The Welfare Costs of Business Cycles Unveiled: Measuring the Extent of Stabilization Policies*

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Abstract

How can we measure the welfare benefit of ongoing stabilization policies? We develop a methodology to calculate the welfare cost of business cycles taking into account that observed consumption is partially smoothed. We propose a decomposition that disentangles consumption into a mix of laissez-faire and riskless components, decoupled by a parameter that captures the span of stabilization power. We estimate this parameter profiting from two distinct variance regimes measured in the historical consumption data and an identification strategy for the mapping between the span for each of these periods. In our preferred specification, we find that the welfare cost of total fluctuations is 11 percent of lifetime consumption, of which 82 percent is smoothed by the status quo policies, yielding a residual 1.8 percent of consumption to be tackled by policymakers.

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1 Introduction

In a tough challenge to conventional wisdom, Lucas (1987) asked how much Americans would be willing to pay, in terms of consumption, to live in an economy that is not subject to the macroeconomic volatility that the US witnessed during the post-war period. Finding that a representative consumer would sacrifice at most one-tenth of a percent of lifetime consumption, Lucas concluded that there would be little benefit in further attempting to stabilize the residual risk of business cycles.

Lucas's model assumed a representative agent with constant relative risk aversion (CRRA) instantaneous utility, whose consumption sequence is represented by a trend-stationary process with i.i.d. shocks. Not surprisingly, this seminal result attracted a great deal of controversy and generated a wealth of literature that revisits his estimates. One branch, for example, expanded the analysis to heterogeneous agents' economies (İmrohoroğlu, 1989; Krusell and Smith Jr., 1999; De Santis, 2007; Krusell et al., 2009). Another branch of the literature follows closely Lucas' approach, assuming a stochastic process for consumption, but moving from trend-stationary to difference stationary processes (Obstfeld, 1994; Issler et al., 2008; Reis, 2009; Guillén et al., 2014; Barros Jr et al., 2023).

This paper fits into the second branch of the literature and addresses a general issue, subtly present in Lucas (1987), that calls for a new measurement effort when estimating the welfare costs of business cycles: all observed consumption is already partially smoothed. That is, the data we gather for consumption stem from a realized allocation that is subject to the status quo of economic stabilization policies. To do so, we propose a tractable decomposition that allows us to compute the reach of the ongoing stabilization policies, devise an identification strategy to connect the model to the data and estimate the parameters of the decomposition, and finally, measure the total cost of economic fluctuations in this context.

Lucas's (1987) approach to measuring the welfare costs of business cycles is grounded in the comparison of the consumer's well-being with the observed consumption sequence,

$\{C_t\}_{t=0}^{\infty}$, and with its smoothed counterfactual version, $\{\bar{C}_t\}_{t=0}^{\infty}$.[1] Recognizing that observed consumption is partially smoothed, this would be the welfare cost of residual fluctuations. If we estimate an alternative counterfactual consumption sequence, $\{\tilde{C}_t\}_{t=0}^{\infty}$ or laissez-faire consumption, that would prevail under minimal macroeconomic stabilization policies and interventions, we could measure the welfare benefits of the ongoing stabilization policies by comparing the the improvement in the consumer's well-being from $\{\tilde{C}_t\}_{t=0}^{\infty}$ to $\{C_t\}_{t=0}^{\infty}$. We then drew inspiration from the approach in Alvarez and Jermann (2004) and modeled observed consumption as a weighted geometric mean of the laissez-faire and the riskless consumption: $C_t(\theta) \equiv \bar{C}_t^{\theta}\tilde{C}_t^{1-\theta}$, where the parameter θ measures the degree to which consumption has already been smoothed. As a result, our approach allows us to map the stabilization policies to a single parameter θ, which we named the *span of stabilization power*.

Assuming the CRRA instantaneous utility and a general log-normal form for the laissez-faire consumption, we put forward our main contribution: a decomposition in which the the welfare cost of total economic fluctuations (consume $\tilde{C}_t$ rather than $\bar{C}_t$) can be disentangled into the benefit of ongoing policies (consume $C_t(\theta)$ rather than $\tilde{C}_t$) and the cost of residual fluctuations (consume $C_t(\theta)$ rather than $\bar{C}_t$). To apply this decomposition we need to both specify the consumption process and identify θ. We dialogue directly with the classic literature by using three log-normal consumption processes: the one of Lucas (1987) with transitory shocks, the one of Obstfeld (1994) with permanent innovations,[2] and a third one that departs from the i.i.d. structure and uses an ARIMA process for the consumption series as proposed by Reis (2009), which we are able to incorporate into our framework with the use of the Beveridge-Nelson decomposition (Beveridge and Nelson, 1981; Issler et al., 2008; Guillén et al., 2014).[3]

Regarding the identification of the parameter θ, we resort to the more novel literature of identification in macroeconomics and couple it with the relevant facts of US macroeco-

[1] Lucas (1987) derives this riskless consumption sequence by applying the unconditional expectation operator to the assumed consumption process, i.e., $\bar{C}_t \equiv \mathbb{E}[C_t]$.

[2] Obstfeld (1994) posited a random walk process, implying that its detrended version (first difference) has no serial correlation. The detrended consumption process proposed by Lucas (1987) has the same features.

[3] Notice that we employ these processes to model the laissez-faire consumption, while the literature has used them for the observed consumption.

nomic history. Our choice of data is an augmented version of the historical consumption series provided by Barro and Ursúa (2010), which shows a significant decrease in volatility after WWII. Such a pattern is identified and confirmed by (i) the established literature on the topic; (ii) the visual inspection of the data; and (iii) a statistical test (ICSS) that finds structural breaks in the variance of time series, which points to 1947 as the only year in our sample when such a break occurs.

These three pieces of evidence allow us to design our identification strategy: we divide the sample into pre- and post-war periods with distinct measured volatilities, and thus two θ's, attributing the difference between them to the larger role and presence of stabilization policies in the second period. We then assume that the laissez-faire consumption volatility remains unchanged during the whole sample and that the span of stabilization policies in the first period, θ_1, can be considered as given at a low level due to the incipient presence of stabilization policies in the pre-war period.[4] Such a discontinuity-based strategy enables us to pin down the span of stabilization policies from 1947 until today conditional on θ_1, i.e., the mapping $\theta_2(\theta_1)$, for $\theta_1 \in [0, 1]$. We can then use this mapping to estimate $\hat{\theta}_2$ and plug it into our decomposition of the welfare costs of total economic fluctuations.

We estimated all three aforementioned consumption processes, but our preferred specification is the one stemming from the ARIMA process, which, among the three considered, best models and fits the time series of consumption. The first difference of the series follows an AR(1) process after 1947, making it straightforward to use the Beveridge-Nelson decomposition to obtain our estimates. We find that the span of stabilization policies, $\hat{\theta}_2$, already smoothed 61 to 73 percent of the laissez-faire consumption shocks in the post-war period, depending on the chosen value of θ_1 within a grid spanning zero to 30 percent of pre-war stabilization.

Our identification strategy and statistical testing essentially reduce structural changes in the economy after 1947 to a unique change of value in θ that remains constant until the final period of the sample. However, there are several potential explanations for the

[4] In our empirical approach we consider a set with distinct possible values for θ_1 defined at a chosen grid.

observed reduction in post-war consumption volatility that could lie beyond the overarching umbrella of a unique paradigm shift of stabilization policies. In order to consider such alternative explanations and allow more flexibility to our approach, we also estimate a time-varying θ. Inspired by Stock and Watson (2007), we use an approach analogous to their exercise on changes in the post-war univariate inflation process and estimate a process with stochastic volatility for our consumption series after 1947. With this methodology, we are able to recover a stochastic $\hat{\theta}_{2,t}$ for the period, relaxing part of our identification strategy. We find that the estimated time-varying span smooths consumption in a range that closely gravitates around our initial estimate for the post-1947 period, with values between 65 and 77 percent.

Given the closeness of the time-varying θ estimates to the values obtained in our initial two-period approach, we are able to return to them and select one of the initially estimated values for $\hat{\theta}_2$. This then allows us to use our theoretical decomposition, plug in the estimated values, and compute the different welfare costs. We find the total cost of economic fluctuations to be 11 percent of lifetime consumption. Close to 82 percent of such costs are already covered by stabilization policies, yielding that more than 9 percent of the smoothed lifetime consumption is left unaccounted for if one does not take into account the benefit of ongoing stabilization policies.[5] Since the residual 1.8 percent of the costs still to be smoothed is the easiest measure to compare with the value that would be implied by the literature in our framework - a calculation of the costs that only takes into account the measured variance of consumption -, we are able to find a residual cost that is two times higher than the usual numbers even when taking into account that observed consumption is partially smoothed.

In order to check the robustness of our analyses, we tackle the possibility that the log-consumption series has a structural break that we should consider beyond the one identified in its volatility. We conduct a Bai-Perron test (Bai and Perron, 2003) and find that there is one break in the first difference of log-consumption in 1934. We adjust the

[5]The large welfare benefits of ongoing policies were, in fact, anticipated in Otrok (2001a): "The estimates provided here, and elsewhere, are aimed at measuring the gains from removing the residual risk. The gains to moving from a regime in which there are no efforts to smooth aggregate volatility to the current regime may be very large."

sample, run the same regression, and find a decrease of only 1 percentage point in $\hat{\theta}_2$, reinforcing our initial findings.[6]

Roadmap The paper is organized as follows. The next section reviews the literature and discusses our contribution. Section 3 describes the model and lays out our theoretical results. Section 4 applies the results of the previous sections to three different applications. Section 5 outlines our empirical approach and describes our identification strategy. Section 6 shows our estimation results and an exercise with a time-variant θ. Section 7 uses the estimates and shows the computed results for welfare costs. Section 8 discusses a robustness exercise on structural breaks. Finally, Section 9 concludes the paper.

2 Related Literature

To measure how uncertainties related to business cycles affect risk-averse consumers, Lucas (1987) proposed a framework in which society is represented by a representative agent. The agent's lifetime utility depends on a constant relative risk aversion (CRRA) instantaneous utility, and the consumption sequence is represented by a trend-stationary process with i.i.d. shocks. In analyzing the US case, Lucas concluded that further attempts to stabilize the residual risk of business cycles would yield a negligible benefit.

Given the simplified assumptions made by Lucas (1987), the literature has cast doubt on his conclusions.[7] One branch departs from the representative agent setting and estimates the welfare costs of business cycles under incomplete markets and heterogeneous agents, as seen in İmrohoroğlu (1989), Krusell and Smith Jr. (1999), Storesletten et al. (2001), De Santis (2007), and Krusell et al. (2009). For instance, the latter concluded that the average gain from eliminating cycles is as much as 1 percent in consumption equivalents.

However, another branch of the literature maintains the representative agent approach

[6]We conduct further robustness analyses on the estimation procedure of θ_2. In Appendix D, we adjust the sample used in the regression with the removal of the inter-war period and also with the original data sample by Barro and Ursúa (2010). The results are similar and consistent with our main analysis.

[7]For an in-depth early discussion of this literature, see Barlevy (2005).

but questions other assumptions made by Lucas. For instance, Obstfeld (1994) switches the original transitory shocks for permanent ones and focuses on their interaction with recursive preferences. Regarding the consumption process, Obstfeld specifically assumed a random walk process.[8] Issler et al. (2008) and Guillén et al. (2014) model consumption using the Beveridge-Nelson decomposition, where consumption depends on both transitory and permanent shocks. Lastly, Reis (2009) models consumption through an ARIMA(p,1,q) process. For the preferred ARIMA model, he concluded that the welfare cost of business cycles in the US economy ranges from 0.31 percent to 0.94 percent when the relative risk aversion coefficient equals one.

Part of the literature discusses which preferences should be adopted. Besides the recursive preferences explored initially by Obstfeld (1994), the literature also investigated time-non-separable and first-order risk aversion preferences. For instance, Otrok (2001a) allows for time-non-separability, but finds that plausible parameter values yield negligible costs of business cycles. Assuming preferences that exhibit first-order risk aversion, Dolmas (1998) showed that the cost of business cycles can be more than 20 percent of lifetime consumption, when shocks are permanent. However, as discussed by Lucas (2003) and Reis (2009), results like that typically came at the expense of assuming that people are extremely averse to risk, which appears to be inconsistent with the risk-taking we observe in their choices. Not by chance, Otrok (2001b) argued that if one chooses an appropriate preference form, one can find a welfare cost of business cycles as large as one wants.

Our decomposition is based on the CRRA instantaneous utility adopted by Lucas (1987) and subsequent works such as Issler et al. (2008), Reis (2009), and Guillén et al. (2014). For comparison purposes, we employ three different consumption processes. First, we use the trend-stationary process with transitory i.i.d. shocks, which allows us to evaluate our decomposition under the same assumption as Lucas's original work. Second, we consider the random walk process proposed by Obstfeld (1994), enabling an investigation into the impacts of transitioning from transitory to permanent shocks. Third, following the flexible approach of Reis (2009), we fit an ARIMA process for consumption

[8]The random walk process was also employed by Van Wincoop (1994), Dolmas (1998) and Barros Jr et al. (2023).

data and incorporate it into our framework using the Beveridge-Nelson decomposition. This way, we cover the setups of the works most related to ours.

Finally, we acknowledge that other approaches have emerged in the literature to investigate the welfare cost of business cycles. By using asset prices, Alvarez and Jermann (2004) developed an approach that does not require the specification of preferences to measure the welfare cost of consumption fluctuations. More recently, Hai et al. (2020) include memorable goods[9] and Constantinides (2021) focuses on the role of idiosyncratic shocks households face that are unrelated to the business cycle. Jorda et al. (2020) investigate the welfare costs of business cycles taking into account possible disasters.[10] Finally, Dupraz et al. (2019) develop a plucking model of business cycles taking into account the asymmetric nature of economic fluctuations.

Our decomposition emerges as a result of modeling observed consumption as being partially smoothed. The proposed weighted geometric mean of laissez-faire consumption and riskless consumption depends on the weight θ, which we need to identify. For this reason, our paper is also embedded in two other major strands of the literature in macroeconomics: (i) the literature that studies the measurement of historical macroeconomic data; and (ii) the literature of identification in macroeconomics.

We conduct our data analysis grounding it in the literature on macroeconomic history. Our sample is built directly from the historical data compiled by Barro and Ursúa (2010) and when developing our identification strategy, we base it on Barro and Ursúa's (2008) observation that for the OECD economies, there is a change in consumption volatility in the post-war period. Our approach also dialogues with the seminal work of Romer (1986) and Balke and Gordon (1989) that documents the challenges faced when measuring the volatility of macroeconomic aggregates, and we show how our methodology can reconcile improvements in both measurement and stabilization after WWII. Here we add our estimation of the unique structural break in the volatility of consumption in 1947 as measured by the Inclan and Tiao (1994) test that is used in our identification exercise.

[9] A good, as defined in Hai et al. (2020), is "memorable if a consumer draws utility from her past consumption experience."

[10] Other examples in this literature are Barro and Jin (2011) and Gourio (2012).

We also view our work as building on the effort of calculating the costs of business cycles, with critical attention to measurement and identification. We resort to Nakamura et al.'s (2013) insight of using the variation in the volatility of the consumption series to better identify the shift in the role of stabilization policies. Moreover, we build on Nakamura et al. (2017) in our use of both transitory and permanent formulations for the shocks in conjunction with a time-varying volatility for the consumption series. Our paper contributes here by using different methodologies to model the measured time-series aspects of the consumption data. For example, we use the Beveridge-Nelson decomposition to tie back our methodology to its ARIMA components, and also, to the extent of our knowledge, we are the first to connect the Stock and Watson (2007) methodology for the inflation process to its time-varying volatility.

3 Model

3.1 Environment and Definitions

The economy is populated by a representative consumer whose lifetime utility is given by $\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t u(C_t)\right]$, where C_t is consumption in period t, $\beta \in (0,1)$ is an intertemporal discount factor, $u(\cdot)$ is the instantaneous utility function, and $\mathbb{E}_0[\cdot]$ is the expectation operator conditional on the information set $\mathcal{I}_0$.[11] We begin with a few definitions:

Definition 1. *Define $\tilde{C}_t$ as consumption with minimal intervention and incipient stabilization policies. Then $\{\tilde{C}_t\}_{t=0}^{\infty}$ is the laissez-faire consumption sequence.*

Definition 2. *Define $\bar{C}_t \equiv \mathbb{E}_0[\tilde{C}_t]$. Then $\{\bar{C}_t\}_{t=0}^{\infty}$ is the riskless consumption sequence.*

We can now define the welfare cost of the total economic fluctuations as the constant $\lambda^T > 0$ that solves the following condition:

[11] We assume that the expectation is taken before the realization of any uncertainty in period 0, as in some calculations done by Obstfeld (1994) and Reis (2009). In that sense, consumption in that period is treated as a stochastic variable. Under this assumption, we compare the lifetime utility in two worlds: one in which the agent is still uncertain about all consumption flows, and the other in which the entire consumption sequence is known, as in Lucas (1987).

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t u\left((1+\lambda^T)\tilde{C}_t\right)\right]=\sum_{t=0}^{\infty}\beta^t u\left(\bar{C}_t\right). \tag{1}$$

The parameter λ^T measures the constant compensation required by the consumer to be indifferent between the adjusted laissez-faire, $\{(1+\lambda^T)\tilde{C}_t\}_{t=0}^{\infty}$, and the riskless consumption sequences.

Given that the observed time series on consumption is subject to the ongoing stabilization policies, we can view it as the combination of two extreme cases: (i) the (unobserved) consumption series under minimal intervention and incipient stabilization policies, $\tilde{C}_t$, and (ii) the (unobserved) perfectly smoothed consumption, $\bar{C}_t$. We then model the (observed) partially smoothed consumption as a weighted geometric average:

$$C_t\left(\theta\right)\equiv\bar{C}_t^{\theta}\tilde{C}_t^{1-\theta}, \tag{2}$$

where the parameter $\theta\in[0,1]$ measures the degree of consumption smoothing. Thus, θ can be interpreted as the span of the stabilization power of governmental policies. This formulation is inspired by the one used in Alvarez and Jermann (2004), in which the total cost function has useful properties that allow its estimation using asset prices, is directly comparable to the one in Lucas (1987) and is consistent with incomplete markets in a first-order approximation. The weighted geometric average is convenient for deriving analytical expressions for welfare costs, which allow us to characterize them and is valuable for a deeper understanding of the mechanisms behind such costs. Additionally, it implies an intuitive arithmetic average for log consumption, which is the way this variable is commonly written in econometric models.

We can now define the benefit of the ongoing stabilization policies as the constant $\lambda^B>0$ that solves the following condition:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t u\left((1+\lambda^B)\tilde{C}_t\right)\right]=\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t u(C_t(\theta))\right]. \tag{3}$$

The parameter λ^B is the compensation required by the consumer to be indifferent between the adjusted laissez-faire consumption sequence and the effective consumption sequence, $\{C_t(\theta)\}_{t=0}^{\infty}$.

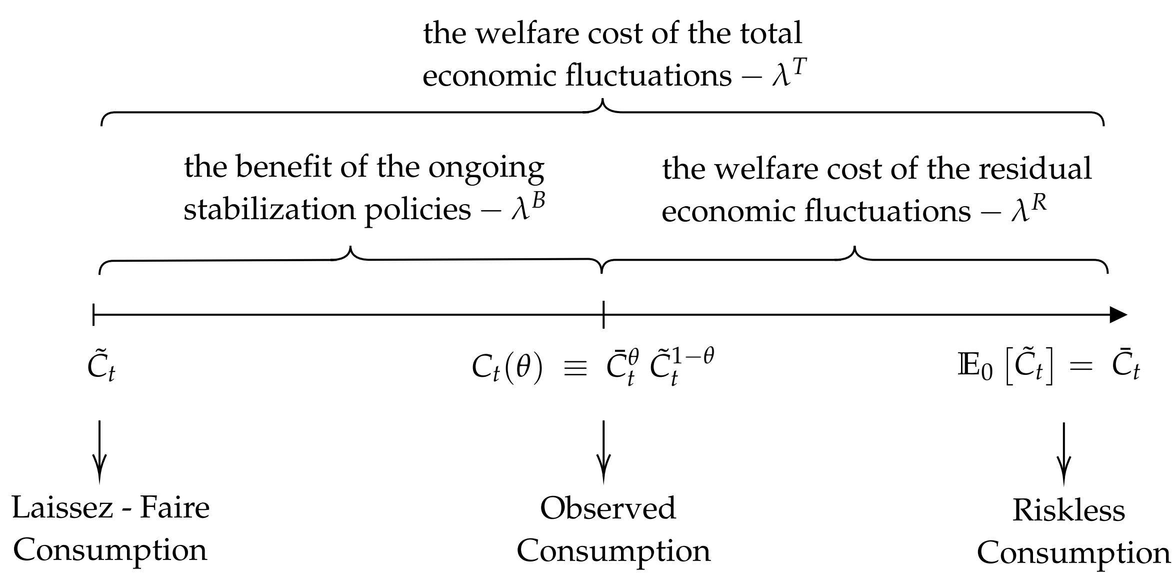
Finally, we can compute what is left to be stabilized by defining the welfare cost of the residual economic fluctuations as the constant $\lambda^R > 0$ that solves the following condition:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t u\left((1+\lambda^R)C_t(\theta)\right)\right] = \sum_{t=0}^{\infty}\beta^t u\left(\bar{C}_t\right). \tag{4}$$

The parameter λ^R measures the constant compensation required by the consumer to be indifferent between the adjusted partially smoothed consumption sequence $\{(1+\lambda^R)C_t(\theta)\}_{t=0}^{\infty}$ and the aforementioned riskless sequence.

Figure 1 summarizes our modelling by showing where each parameter and measure defined is located in a spectrum of consumption that spans the highest to the lowest level of risk.

Figure 1: Decomposition of the welfare cost of the total economic fluctuations



3.2 Assumptions

In order to calculate λ^T, λ^B, and λ^R and guarantee tractability, we assume a log-normal process for $\tilde{C}_t$, which implies that $C_t(\theta)$ is also log-normal.

Assumption 1. *Log-normal consumption process:* $\tilde{C}_t = \alpha_0(1+\alpha_1)^t X_t$, *where* $X_t = e^{x_t - 0.5\sigma_t^2}$, *with* $x_t|\mathcal{I}_0 \sim \mathcal{N}\left(0, \sigma_t^2\right)$.

Following Lucas (1987), we assume a CRRA instantaneous utility with parameter γ:

$$u(C) = \begin{cases} \dfrac{C^{1-\gamma}}{1-\gamma}, \text{ if } \gamma > 1 \\ \ln(C), \text{ if } \gamma = 1 \end{cases} \tag{5}$$

We also need assumptions that guarantee that the sums in conditions (1), (3), and (4) are all finite. They are:

Assumption 2. *The constant* $\Gamma \equiv \beta\left(1+\alpha_1\right)^{1-\gamma} \in (0,1)$.

Assumption 3. $\sum\limits_{t=0}^{\infty} \Gamma^t \exp\left\{-0.5\gamma\left(1-\gamma\right)\sigma_t^2\right\} < \infty$.[12]

Under Assumption 1, riskless consumption is given by $\bar{C}_t = \mathbb{E}_0[\tilde{C}_t] = \alpha_0(1+\alpha_1)^t$ and is deterministic. Furthermore, $\tilde{C}_t = \bar{C}_t X_t$, and the partially smoothed consumption can be rewritten as $C_t\left(\theta\right) = \bar{C}_t X_t^{1-\theta}$. From this formulation it is easy to see that the larger the parameter θ, the less important is the stochastic part of the partially smoothed consumption.

3.3 Theoretical Results

We can now derive closed-form solutions for the parameters λ^B, λ^R and λ^T. Propositions 1, 2, and 3 establish, respectively, each of these parameters. The final step consists of using

[12]Note that $\sum_{t=0}^{\infty} \Gamma^t \exp\left\{-0.5\left(1-\theta\right)\left(1-\gamma\right)\left(\gamma+\theta-\gamma\theta\right)\sigma_t^2\right\} < \sum_{t=0}^{\infty} \Gamma^t \exp\left\{-0.5\gamma\left(1-\gamma\right)\sigma_t^2\right\}$ if $\gamma > 1$. This result ensures that the λ's are finite in some of our results.

the propositions to obtain our main decomposition of the welfare cost of total economic fluctuations. All proofs are shown in Appendix A.

Proposition 1. *Under Assumptions 1 and 3 the benefit of the ongoing stabilization policies is given by*

$$\lambda^B = \begin{cases} \exp\left\{\theta \frac{1-\beta}{2} \sum_{t=0}^{\infty} \beta^t \sigma_t^2\right\} - 1, & \text{if } \gamma = 1 \\ \left[\frac{\sum_{t=0}^{\infty} \Gamma^t \exp\left\{-0.5(1-\gamma)(1-\theta)(\theta+\gamma-\gamma\theta)\sigma_t^2\right\}}{\sum_{t=0}^{\infty} \Gamma^t \exp\left\{-0.5\gamma(1-\gamma)\sigma_t^2\right\}}\right]^{\frac{1}{1-\gamma}} - 1, & \text{if } \gamma > 1 \end{cases} \tag{6}$$

Proposition 2. *Under Assumptions 1, 2, and 3 the welfare cost of the residual macroeconomic fluctuations is given by*

$$\lambda^R = \begin{cases} \exp\left\{(1-\theta) \frac{1-\beta}{2} \sum_{t=0}^{\infty} \beta^t \sigma_t^2\right\} - 1, & \text{if } \gamma = 1 \\ \left[\frac{\sum_{t=0}^{\infty} \Gamma^t}{\sum_{t=0}^{\infty} \Gamma^t \exp\left\{-0.5(1-\gamma)(1-\theta)(\theta+\gamma-\gamma\theta)\sigma_t^2\right\}}\right]^{\frac{1}{1-\gamma}} - 1, & \text{if } \gamma > 1 \end{cases} \tag{7}$$

Proposition 3. *Under Assumptions 1, 2, and 3 the welfare cost of the total economic fluctuations is given by*

$$\lambda^T = \begin{cases} \exp\left\{\frac{1-\beta}{2} \sum_{t=0}^{\infty} \beta^t \sigma_t^2\right\} - 1, & \text{if } \gamma = 1 \\ \left[\frac{\sum_{t=0}^{\infty} \Gamma^t}{\sum_{t=0}^{\infty} \Gamma^t \exp\left\{-0.5\gamma(1-\gamma)\sigma_t^2\right\}}\right]^{\frac{1}{1-\gamma}} - 1, & \text{if } \gamma > 1 \end{cases} \tag{8}$$

We can now state our main result in Theorem 1 below: the decomposition of the welfare cost of total economic fluctuations.

Theorem 1. *Under Assumptions 1 to 3 and CRRA utility (5), there is a decomposition of the welfare cost of total economic fluctuations in the form*

$$1 + \lambda^T = \left(1 + \lambda^B\right)\left(1 + \lambda^R\right). \tag{9}$$

4 Applications

In this section we characterize λ^T, λ^B, and λ^R using three different shock structures for the consumption process: the classic ones of Lucas (1987) with transitory shocks and of Obstfeld (1994) with permanent shocks, and one with an ARIMA process for consumption as proposed in Reis (2009) using the Beveridge-Nelson (BN) decomposition (Beveridge and Nelson, 1981; Issler et al., 2008; Guillén et al., 2014). The details of all calculations are shown in Appendix A.1.

Example 1 - Transitory Shocks (Lucas, 1987): Define $C_t = \alpha_0(1+\alpha_1)^t e^{-0.5\sigma_\varepsilon^2 + x_t^L}$, where $x_t^L|\mathcal{I}_0 \sim \mathcal{N}(0, \sigma_\varepsilon^2)$. Hence,

$$
\lambda^T = \begin{cases} \exp\left\{\frac{1}{2}\sigma_\varepsilon^2\right\} - 1, & \text{if } \gamma = 1 \\ \exp\left\{\frac{\gamma}{2}\sigma_\varepsilon^2\right\} - 1, & \text{if } \gamma > 1 \end{cases} \tag{10}
$$

$$
\lambda^B = \begin{cases} \exp\left\{\frac{\theta}{2}\sigma_\varepsilon^2\right\} - 1, & \text{if } \gamma = 1 \\ \exp\left\{\frac{\gamma}{2}\sigma_\varepsilon^2 - \frac{1}{2}(1-\theta)(\theta+\gamma-\gamma\theta)\sigma_\varepsilon^2\right\} - 1, & \text{if } \gamma > 1 \end{cases} \tag{11}
$$

$$
\lambda^R = \begin{cases} \exp\left\{\frac{1-\theta}{2}\sigma_\varepsilon^2\right\} - 1, & \text{if } \gamma = 1 \\ \exp\left\{\frac{1}{2}(1-\theta)(\theta+\gamma-\gamma\theta)\sigma_\varepsilon^2\right\} - 1, & \text{if } \gamma > 1 \end{cases} \tag{12}
$$

For this process, the variance in Assumption 1 becomes $\sigma_t^2 = \sigma_\varepsilon^2$. Consequently, Assumption 3 is satisfied as long as Assumption 2 holds.

Example 2 - Permanent Shocks (Obstfeld, 1994): Define $C_t = \alpha_0(1+\alpha_1)^t e^{-0.5\sigma_\varepsilon^2 + x_t^O}$, where $x_t^O = \sum_{i=0}^{t} \varepsilon_i$, $\varepsilon_i|\mathcal{I}_0 \sim \mathcal{N}(0, \sigma_\varepsilon^2)$.[13] Thus,

[13] In some calculations, Obstfeld (1994) treats C_0 as known. We consider the case where the expectation is taken before the realization of the shock ε_0.

$$\lambda^T = \begin{cases} \exp\left\{\frac{1}{2}\frac{1}{1-\beta}\sigma_\varepsilon^2\right\} - 1, & \text{if } \gamma = 1 \\ \exp\left\{0.5\gamma\sigma_\varepsilon^2\right\}\left[\frac{1-\Gamma\exp\left\{-0.5\gamma(1-\gamma)\sigma_\varepsilon^2\right\}}{1-\Gamma}\right]^{\frac{1}{1-\gamma}} - 1, & \text{if } \gamma > 1 \end{cases} \tag{13}$$

$$\lambda^B = \begin{cases} \exp\left\{\theta\frac{1}{2}\frac{1}{1-\beta}\sigma_\varepsilon^2\right\} - 1, & \text{if } \gamma = 1 \\ \frac{\exp\left\{0.5\gamma\sigma_\varepsilon^2\right\}}{\exp\left\{0.5(1-\theta)[\gamma+\theta-\theta\gamma]\sigma_\varepsilon^2\right\}}\left[\frac{1-\Gamma\exp\left\{-0.5\gamma(1-\gamma)\sigma_\varepsilon^2\right\}}{1-\Gamma\exp\left\{-0.5(1-\gamma)(1-\theta)(\theta+\gamma-\gamma\theta)\sigma_\varepsilon^2\right\}}\right]^{\frac{1}{1-\gamma}} - 1, & \text{if } \gamma > 1 \end{cases} \tag{14}$$

$$\lambda^R = \begin{cases} \exp\left\{(1-\theta)\frac{1}{2}\frac{1}{1-\beta}\sigma_\varepsilon^2\right\} - 1, & \text{if } \gamma = 1 \\ \frac{1}{\exp\left\{0.5(1-\theta)[\gamma+\theta-\theta\gamma]\sigma_\varepsilon^2\right\}}\left[\frac{1-\Gamma\exp\left\{-0.5(1-\gamma)(1-\theta)(\theta+\gamma-\gamma\theta)\sigma_\varepsilon^2\right\}}{1-\Gamma}\right]^{\frac{1}{1-\gamma}} - 1, & \text{if } \gamma > 1 \end{cases} \tag{15}$$

In this case, $\sigma_t^2 = Var_0\left[\sum_{i=0}^t \varepsilon_i\right] = (t+1)\sigma_\varepsilon^2$, and the condition $\Gamma\exp\{-0.5\gamma(1-\gamma)\sigma_\varepsilon^2\} < 1$ is sufficient for Assumption 3 to be valid.

Example 3 - ARIMA-BN Process (Reis, 2009): Define

$$C_t = \alpha_0(1+\alpha_1)^t \exp\left\{-\frac{1}{2}\sigma_{x_t^{BN}}^2\right\}\exp\left\{x_t^{BN}\right\} \tag{16}$$

where, to obtain x_t^{BN}, we apply the Beveridge-Nelson decomposition. We follow these steps:

1. Given a process, $C_t = f(t) + u_t$, where $f(t)$ is deterministic and $(1-L)u_t = \psi(L)\varepsilon_t$, with $\psi(L) = \sum_{j=0}^\infty \psi_j L^j$. Define $\varphi_j = -\sum_{i=j+1}^\infty \psi_i$.
2. Then, $x_t^{BN} = \psi(1)\sum_{j=0}^t \varepsilon_j + \sum_{j=0}^t \varphi_j\varepsilon_{t-j}$, with $\varepsilon_j|\mathcal{I}_0 \sim \mathcal{N}(0,\sigma_\varepsilon^2)$.
3. We follow Issler et al. (2008) and rewrite $\sigma_{x_t^{BN}}^2$ as $\tilde{\sigma}_{x_t^{BN}}^2 = \rho_0 + \rho_1 t$, where

$$\rho_0 \equiv \psi(1)^2\sigma_\varepsilon^2 + 2\psi(1)\sum_{j=0}^\infty \varphi_{t-j}\sigma_\varepsilon^2 + \sum_{j=0}^\infty \varphi_{t-j}^2\sigma_\varepsilon^2 \quad \text{and} \quad \rho_1 \equiv \psi(1)^2\sigma_\varepsilon^2 \tag{17}$$

4. Hence, since we know $x_t^{BN} \sim N\left(0,\sigma_{x_t^{BN}}^2\right)$ and can approximate $\sigma_{x_t^{BN}}^2$ with $\tilde{\sigma}_{x_t^{BN}}^2$.

We can then compute the λ's for this structure of shocks:[14]

$$\lambda^T = \begin{cases} \exp\left\{\frac{1}{2}\left(\rho_0 + \frac{\beta}{1-\beta}\rho_1\right)\right\} - 1, & \text{if } \gamma = 1 \\ \exp\left\{0.5\gamma\rho_0\right\}\left[\frac{1-\Gamma\exp\{-0.5\gamma(1-\gamma)\rho_1\}}{1-\Gamma}\right]^{\frac{1}{1-\gamma}}, & \text{if } \gamma > 1 \end{cases} \tag{18}$$

$$\lambda^B = \begin{cases} \exp\left\{\frac{\theta}{2}\left(\rho_0 + \frac{\beta}{1-\beta}\rho_1\right)\right\} - 1, & \text{if } \gamma = 1 \\ \frac{\exp\{0.5\gamma\rho_0\}}{\exp\{0.5(1-\theta)(\theta+\gamma-\gamma\theta)\rho_0\}} \times & \\ \quad \times \left[\frac{1-\Gamma\exp\{-0.5\gamma(1-\gamma)\rho_1\}}{1-\Gamma\exp\{-0.5(1-\gamma)(1-\theta)(\theta+\gamma-\gamma\theta)\rho_1\}}\right]^{\frac{1}{1-\gamma}} - 1, & \text{if } \gamma > 1 \end{cases} \tag{19}$$

$$\lambda^R = \begin{cases} \exp\left\{\frac{1-\theta}{2}\left(\rho_0 + \frac{\beta}{1-\beta}\rho_1\right)\right\} - 1, & \text{if } \gamma = 1 \\ \exp\left\{0.5\left(1-\theta\right)\left(\theta+\gamma-\gamma\theta\right)\rho_0\right\} \times & \\ \quad \times \left[\frac{1-\Gamma\exp\{-0.5(1-\gamma)(1-\theta)(\theta+\gamma-\gamma\theta)\rho_1\}}{1-\Gamma}\right]^{\frac{1}{1-\gamma}} - 1, & \text{if } \gamma > 1 \end{cases} \tag{20}$$

In this case, $\Gamma\exp\{-0.5\gamma(1-\gamma)\rho_1\} < 1$ is sufficient for Assumption 3 to be valid.

5 Empirical Approach

In order to estimate the parameters of the underlying consumption process and compute λ^T, λ^B and λ^R, we have to be specific in our assumptions about the structure of the shocks. Following the examples in the last section, we consider the three aforementioned cases: transitory, permanent, and ARMA shocks. In this section, we first develop the regressions to be estimated in the data and characterize the challenge present in the identification of the span of stabilization power, θ, as well as the other parameters of our setting. We then present algebraically and visually the strategy we implement to overcome this difficulty, allowing us to pin down the values to be used in our results.

[14] See some omitted calculations for the steps above and the characterization of the λ's in Appendix A.3.

5.1 Estimation

5.1.1 Transitory Shocks

Assuming transitory shocks under Assumption 1 and applying the logarithm to both sides of equation (2), we have that:

$$\log\left(C_t(\theta)\right) = \log\left(\alpha_0\right) - (1-\theta)0.5\sigma_\varepsilon^2 + t\log\left(1+\alpha_1\right) + (1-\theta)\varepsilon_t. \tag{21}$$

We can reinterpret (21) as a time-series regression of log-per capita consumption c_t with coefficients π_0 and π_1, and error u_t:

$$\log(c_t) = \pi_0 + \pi_1 t + u_t, \tag{22}$$

Note that an identification problem arises when we try to estimate the parameters in equation (21) since $(\alpha_0, \theta, \sigma_\varepsilon^2)$ are all simultaneously mapped to π_0. Furthermore, σ_ε^2 is scaled by $(1-\theta)$, which lies in the background of u_t. Only parameter α_1 is well-identified and can be directly inverted from the estimates since $\alpha_1 = \exp\left(\pi_1\right) - 1$.

5.1.2 Permanent Shocks

Considering the case where permanent shocks hit consumption, we have that:

$$\log\left(C_t(\theta)\right) = \log\left(\alpha_0\right) - (1-\theta)0.5t\sigma_\varepsilon^2 + t\log\left(1+\alpha_1\right) + (1-\theta)\sum_{i=0}^{t}\varepsilon_i. \tag{23}$$

Taking first differences,

$$\Delta\log\left(C_t(\theta)\right) = \log\left(1+\alpha_1\right) - (1-\theta)0.5\sigma_\varepsilon^2 + (1-\theta)\varepsilon_t. \tag{24}$$

We can rewrite equation (24) as:

$$\Delta \log(c_t) = \pi_0 + u_t. \tag{25}$$

The same identification issue arises: $(\alpha_1, \theta, \sigma_\varepsilon^2)$ are behind π_0 with σ_ε^2 scaled by $(1-\theta)$.

5.1.3 ARIMA-BN Process

Similarly, we have that:

$$\Delta \log C_t(\theta) = \log(1+\alpha_1) - 0.5\rho_1 + (1-\theta)\,\Delta x_t^{BN} \tag{26}$$

Hence,

$$\Delta \log C_t(\theta) = \log(1+\alpha_1) - 0.5\psi(1)^2 \sigma_\varepsilon^2 + \psi(L)\,\tilde{\varepsilon}_t \tag{27}$$

where $\tilde{\varepsilon}_t \sim N\left(0, (1-\theta)^2 \sigma_\varepsilon^2\right)$.

Here we use the fact that the per capita consumption series has a unit root and its first difference is stationary.[15] Hence, we can switch to the ARMA(p,q) form:

$$\Phi(L)\,\Delta \log C_t(\theta) = \Phi(1)\left[\log(1+\alpha_1) - 0.5\psi(1)^2 \sigma_\varepsilon^2\right] + \Theta(L)\,\tilde{\varepsilon}_t \tag{28}$$

At this step we estimate an ARMA(p,q) with an intercept for the first difference of the observed log-consumption series. After that, we have $\Phi(L)$ and $\Theta(L)$ and invert the autoregressive lag polynomial to obtain:

$$\Delta \log C_t(\theta) = \left[\log(1+\alpha_1) - 0.5\psi(1)^2 \sigma_\varepsilon^2\right] + \psi(L)\,\tilde{\varepsilon}_t \tag{29}$$

[15] The series is I(1) as identified by the ADF, PP, and DF-GLS tests.

where $\psi(L) = \Theta(L)/\Phi(L)$ was obtained in the estimation process, yielding us $\phi(1)$ and $\psi(1)$. This leaves us again with the scaling factor $(1-\theta)$ in the way of the identification of the process parameters.

5.2 Identification

From our previous characterization of the identification problem, we observed that the scaling of the structural parameters by θ means that the consumption series is partially smoothed due to the ongoing stabilization policies. This means that if we knew θ (or σ_ε^2) in advance, it would be possible to recover all parameters in our consumption model by running a simple regression like the ones shown previously. Since this is not possible, we need to design an identification strategy.

Our strategy consists of exploring an observed variation in the volatility of the historical consumption series in order to identify θ. More precisely, we profit from this variation to identify the mapping induced by distinct measurements to different time-dependent values of θ. We use a combination of three pieces of evidence: (i) the empirical fact documented in the literature that per capita consumption in the US became less volatile after WWII; (ii) a visual analysis in which we plot the series and observe a potentially unique, sharp break in the graph coinciding with the post-war period; and (iii) a statistical result in which we conduct a test to find any breaks in the variance series.

To apply this strategy in the data, we need to use a long series of consumption for the US. Our choice is to build on the data by Barro and Ursúa (2010). This database contains annual observations of US per capita consumption between 1834 and 2009. We complete the sequence of consumption between 2010 and 2019, maintaining their methodology and using the series available from the BEA's NIPA. Finally, we set the data in real terms to 2012.[16]

For the first aforementioned piece of evidence, we follow Lucas (1987), Barro and

[16] We use the series "Personal Consumption Expenditures" in Table 1.1.5, the price index series for the same category in Table 1.1.4, and the series"Population (midperiod thousands)" in Table 2.1 (US Bureau of Economic Analysis, 2021a,b,c).

Ursúa (2008), and Nakamura et al. (2017), who discuss and document the fact that the end of the Second World War marks a substantial decrease in the volatility of consumption over time, exhibiting a heteroskedastic pattern. The second piece of evidence is the visual analysis. In Figure 2, we can observe a sharp and discontinuous break in the variance of consumption after WWII. For the third factor, we apply the iterated cumulative sums of squares (ICSS) algorithm developed by Inclan and Tiao (1994) to detect breaks in the variance of consumption growth. We use a 5 percent significance level to test for multiple breaks.[17] The ICSS algorithm identifies only one break in the variance of consumption growth indicating a sudden decrease in the volatility of consumption growth after 1947.

The three pieces of evidence are then suggestive of a unique and discontinuous break in the variance of consumption, essentially dividing the time series into two distinct regimes, divided by the end of WWII. These thus allow us to make use of a discontinuity-based type of identification strategy, as discussed in Nakamura and Steinsson (2018). Such approaches use a discontinuous change in a series as a way to identify the underlying causal effect or parameter behind that observed change. In our exercise, we then use the unique and discontinuous change in the observed variance of consumption to pindown θ and quantify the consumption-smoothing power of stabilization policies. The main underlying assumption of this strategy is that no other factors, aside from the changes in stabilization policies affect the consumption series of the US change discontinuously at the end of WWII. We revisit this assumption in a detailed discussion below.

To better understand our identification strategy, it is useful to discuss it in formal terms: suppose that we have two periods of time, 1 and 2, and that $Var(\varepsilon_t) = \sigma^2_\varepsilon$ in both periods, but we observe a lower volatility in consumption in period 2. All else constant, we can attribute this difference in the measured volatility to a different span of stabilization power of policies in those periods. To see that, let θ_i and $\hat{\sigma}^2_{u,i}$ be, respectively, the stabilization power and the estimated variance of u_t in period $i \in \{1, 2\}$. Thus, we

[17] We consider the critical value of 1.30 reported in Table 1 of Inclan and Tiao (1994) for a sample size of 200, which is the number closest to our sample. Considering the asymptotic value for the test (1.358) does not change our results.

have that $\hat{\sigma}^2_{u,i} = (1-\theta_i)^2\sigma^2_\varepsilon$. If we knew θ_1 in advance, we could pin down θ_2 using the following identifying equation:

$$\hat{\theta}_2 = 1 - (1-\theta_1)\sqrt{\frac{\hat{\sigma}^2_{u,2}}{\hat{\sigma}^2_{u,1}}}. \tag{30}$$

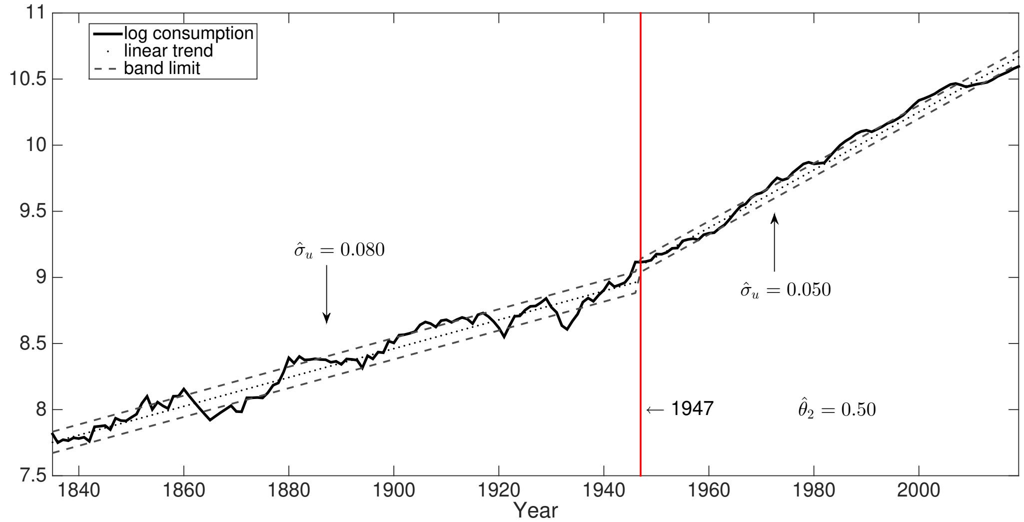
Hence, this strategy allows us to identify the mapping $\theta_2(\theta_1)$, for $\theta_1 \in [0,1]$. The remaining parameter to delineate in the strategy is θ_1. For that, a natural candidate would be a period for which we have information regarding θ_1, such as a period of incipient stabilization policies, i.e., one in which θ_1 is not very large.

In Figure 2, we show our identification strategy at work in the plot of the historical series of consumption. The top panel (2a) shows the series in its log level for the identification with transitory shocks and the bottom panel (2b) shows the series' first difference to accommodate the permanent shocks and ARIMA-BN process approaches as shown in equations (24) and (29).[18]

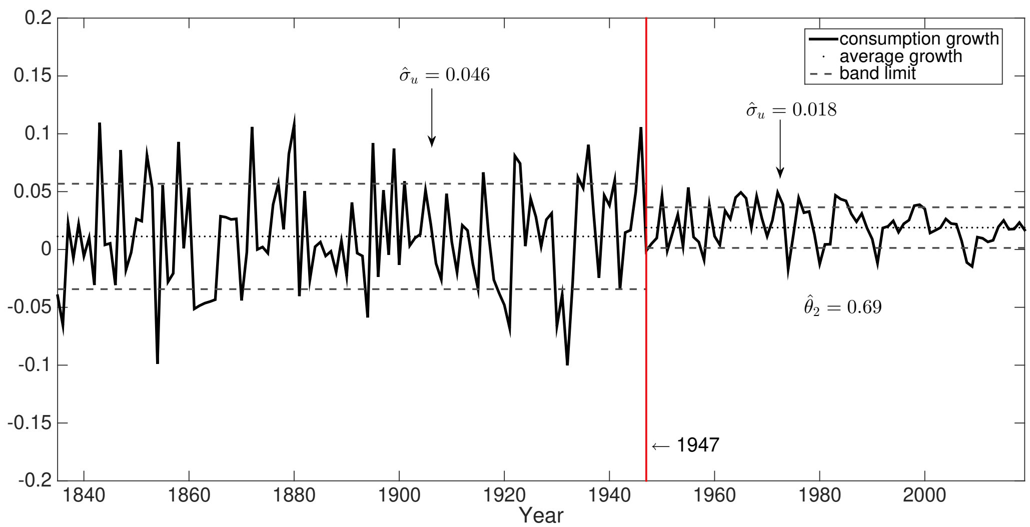
If we divide the series into two periods, pre- and post-war, there is a substantial decrease in the measured standard error after 1947. Focusing on the series with first differences in the bottom panel, for the period between 1835 and 1946, we have that $\hat{\sigma}_u = 0.046$, which then suffers a sharp decrease of more than 60 percent of its value, to $\hat{\sigma}_u = 0.018$, after WWII until today.[19] With such a discontinuous decrease in the volatility of the series, we can plug these measures into equation (30) and, assuming $\theta_1 = 0.20$, for instance, we find that $\hat{\theta}_2 = 0.69$. This indicates a share of 69 percent of smoothed consumption in the observed series post-1947.

[18]In Appendix B.1, we plot in Figure A.1 the visual identification for transitory consumption shocks.
[19]The detailed estimation is shown in Section 6.1 below.

Figure 2: Time series of per capita consumption for the US between 1835 and 2019.



(a) Time series of log consumption.



(b) Time series of consumption growth.

Notes: The figure shows the time series for per capita consumption for the US between 1835 and 2019 with our augmented sample of the Barro and Ursúa (2010) data. There are two panels: the top one uses the series in log levels and the second in growth. The vertical line marks the year 1947, at the end of WWII. We report the standard errors for the two sub-periods generated by this line along with the average and band limits equivalent to $2\sigma_u$. In Appendix B.1, we include Figure A.1 showing the visual identification of the data in panel 2a in an equivalent format to the data in panel 2b.

5.3 Discussion on the Identification Strategy

Our identification strategy relies on a few important assumptions, some of those concerning the formal aspects of our thought experiment and others more focused on the historical details and context of our data and sample. One critical assumption is the division of the sample into only two main periods and its two associated θ's, as well as potential time changes on σ_ε^2. Below we add more references to our discussion of the literature on macroeconomic history that motivates in part our analysis and consideration of incipient stabilization pre-WWII. On top of that, we present our estimations in Section 6.1 over a grid for different values of θ_1, allowing for some sensitivity analysis, and compare the estimates of the variance obtained from a different regression specification that includes the change in the composition of aggregate consumption over time. We also take a deeper dive and relax the assumption of a fixed θ_2 and σ_ε^2 and estimate a time-varying span of stabilization power in Section 6.2.

With respect to the data, we conduct a robustness check aimed at tackling structural breaks in the consumption series and discuss it in detail in Section 8. Also in Section 6.1, we conduct a robustness exercise on the estimation of the variance of consumption used in our identification and estimates, controlling for the secular change in the composition consumption and its shift toward a higher share of services in the aggregate bundle. We also analyze the effects of changes in the sample such as the removal of the inter-war period or the post-Great Recession period in Appendix D. There are other noteworthy aspects that require further discussion and we address those below.

First, a critical point for our measurement of the decrease in consumption's standard error is the seminal argument by Romer (1986) about the spurious decrease in the unemployment rate's volatility after 1948, which was also emphasized for GNP in Balke and Gordon (1989) and revisited for GDP in the context of OECD economies by Barro and Ursúa (2008). The first differing factor in our approach is that we use the series for consumption collected by the BEA since 1929. On top of that, with our augmented sample of Barro and Ursúa's (2010) data, we add an extra 90 years to the length of Romer's (1986) original sample.

Another relevant consideration is the fact that our methodology allows us enough flexibility for a degree of discretion in the interpretation of the span during the incipient stabilization period. In equation (30), the greater θ_1, the smaller the impact of the volatility ratio in the identification of the second period's span. In that sense, the choice of θ_1 can be made larger not only to reflect a historically motivated share of riskless consumption but also to take into account a certain degree of measurement error that undermined the mapping of such stabilization to the collected data.[20]

A pertinent institutional concern is the fact that the systematic publication of the US national accounts in a format similar to today's NIPA started precisely in July of 1947 as a supplement to the Survey of Current Business (SCB), coinciding with the year where we find a structural break in the series for the variance of consumption growth. This first publication contained data spanning 1929-1946, which were subject to minor revisions and additions in the fifties until a first comprehensive revision in 1965 (US Bureau of Economic Analysis, 2001). With at least 17 years of standardized data prior to the cutoff found by the ICSS in 1947, we understand that the algorithm is not picking an idiosyncratic historical event in the compilation of national account statistics. Furthermore, in work using the same data set, Barro and Ursúa (2008) find that the last matched "disaster" contraction of both consumption expenditure and GDP in the US happened in 1947 and Nakamura et al. (2013) find only one disaster episode during 1930-35, providing more evidence of the switch in the volatility regime after 1947.

Apart from the documentation in the data by Barro and Ursúa (2008) and Nakamura et al. (2017) of the decrease in consumption volatility after WWII, it is also worth revisiting some of the literature in macroeconomic history that supports our identification hypothesis, highlighting the critical role of the Great Depression and the war in reshaping the landscape of macroeconomic stabilization policies. Snowdon and Vane (2005) emphasize that the economic institutions at the end of the 20th century were substantially different from the ones prior to 1929. The authors draw from the "defining moment" analysis in

[20] Here we also develop another subtle point mentioned in Lucas's original analysis. In Lucas (1987), footnote 4, there is a mention of Romer (1986) in which the author acknowledges that his calculations do not incorporate her findings and may rely on the 1930s experience.

the seminal work by Bordo et al. (2007), who identify the Great Depression as the turning point for the birth of active macroeconomic policy and stabilization. DeLong (1997), for example, highlights the role of the Employment Act of 1946 and the post-WWII automatic stabilizers as evidence of the shift in paradigm of fiscal policy, while Calomiris and Wheelock (1998) emphasize the increased independence of the Fed after 1951. Finally, Blanchard (2000) argues that the period prior to 1940 was still an exploratory one in terms of macroeconomic thought, with an integrated framework aimed at stabilization surging only after that.

6 Empirical Results

6.1 Estimation

We run regressions (22) and the versions of (25) for permanent shocks and the ARIMA-BN process and obtain their estimated coefficients as well as the error volatility of the two distinct periods, $\hat{\sigma}^2_{u,i}$. We compute the span of stabilization power, $\hat{\theta}_2(\theta_1)$, for different levels in the grid $\theta_1 \in \{0, 0.1, 0.2, 0.3\}$, to allow different policy efficiencies in the initial period. With these values, we can then directly compute $\hat{\sigma}^2_{\varepsilon} = \hat{\sigma}^2_{u,1}/(1-\theta_1)^2$.

For the remaining parameters, in the case of transitory shocks we have that $\hat{\alpha}_1 = \exp(\hat{\pi}_1) - 1$. For permanent shocks, $\hat{\alpha}_1 = \exp\left(\hat{\pi}_0 + (1-\hat{\theta}_2)0.5\sigma^2_{\varepsilon}\right) - 1$. Finally, for the case of the ARIMA-BN process we have that $\hat{\alpha}_1 = \exp\left(\hat{\pi}_0 + (1-\hat{\theta}_2)0.5\hat{\psi}(1)^2\sigma^2_{\varepsilon}\right) - 1$. Table 1 shows the results of our estimations.

Table 1: Estimated parameters.

Transitory shocks				Permanent shocks				ARIMA-BN			
Estimated parameters				Estimated parameters				Estimated parameters			
	$\hat{\pi}_0$	$\hat{\pi}_1$	$\hat{\sigma}_u^2$		$\hat{\pi}_0$	$\hat{\pi}_1$	$\hat{\sigma}_u^2$		$\hat{\pi}_0$	$\hat{\phi}_1$	$\hat{\sigma}_u^2$
1835 - 1946	7.7417	0.0109	0.0065	1835 - 1946	0.0113	-	0.0021	1835 - 1946	0.0113	-	0.0021
	(.0156)	(.0002)			(.0043)				(.0043)		
1947 - 2019	6.6156	0.0219	0.0025	1947 - 2019	0.0203	-	0.0003	1947 - 2019	0.0189	0.2919	0.0003
	(.0427)	(.0003)			(.0020)				(.0030)	(.1321)	
Implied parameters				Implied parameters				Implied parameters			
θ_1	$\hat{\theta}_2$	$\hat{\alpha}_1$	$\hat{\sigma}_\varepsilon^2$	θ_1	$\hat{\theta}_2$	$\hat{\alpha}_1$	$\hat{\sigma}_\varepsilon^2$	θ_1	$\hat{\theta}_2$	$\hat{\alpha}_1$	$\hat{\sigma}_\varepsilon^2$
0.00	0.3731	0.0222	0.0065	0.00	0.6284	0.0209	0.0021	0.00	0.6142	0.0277	0.0021
0.10	0.4358	0.0222	0.0080	0.10	0.6656	0.0209	0.0026	0.10	0.6528	0.0278	0.0026
0.20	0.4985	0.0222	0.0101	0.20	0.7027	0.0210	0.0032	0.20	0.6914	0.0278	0.0032
0.30	0.5612	0.0222	0.0132	0.30	0.7399	0.0210	0.0042	0.30	0.7300	0.0279	0.0042

Notes: The table displays the estimated parameters obtained by running regression (22) for transitory shocks and regression (25) for permanent shocks and the ARIMA-BN process. The time series used is our sample of the augmented Barro and Ursúa (2010) data with sub-periods divided at 1947. The series is I(1) as identified by the ADF, PP, and DF-GLS tests. The first difference of the series is identified, by both the AIC and the BIC criteria, as an ARMA (0,0) for the pre-1947 period and an ARMA(1,0) for the subsequent years. The implied parameters are obtained using the formulas described in the text and equation (30).

Our preferred shock structure is the one with the ARIMA-BN process, since it is the one that most accurately models the data and allows for a more flexible structure without relying on the i.i.d. assumption of either the level or the first difference of the series. We also focus the discussion on the results associated with our preferred choice of initial span, $\theta_1 = 0.20$, since it allows, as mentioned previously, for a combination of some degree of stabilization power and measurement error in the pre-1947 sample. The estimated span is 0.4985 with transitory shocks, 0.7027 with permanent shocks, and 0.6914 with the ARIMA-BN process.[21]

These results show that the average post-war reach of stabilization policies is far from trivial and more than tripled after WWII. The results naturally vary according to the choice of θ_1, but with moderate sensitivity: had we considered a total absence of stabilization policies in the pre-war sample, i.e., $\theta_1 = 0$, we would have that the post-war smoothing factor would be 61 percent for the ARIMA-BN process. With 30 percent of stabilization pre-WWII, the estimated span for the second period of the sample would increase less than 4 percentage points. Moreover, for all shocks, as we increase the value of θ_1, the implied increase in $\hat{\theta}_2$ is incrementally smaller, further contributing to the robustness of the range estimated. Another feature of our strategy shown in Figure 2 is the strict division of the data in 1947. We relax the 1947 cutoff by conducting robustness checks with different windows of time in Section 8 and Appendix D.

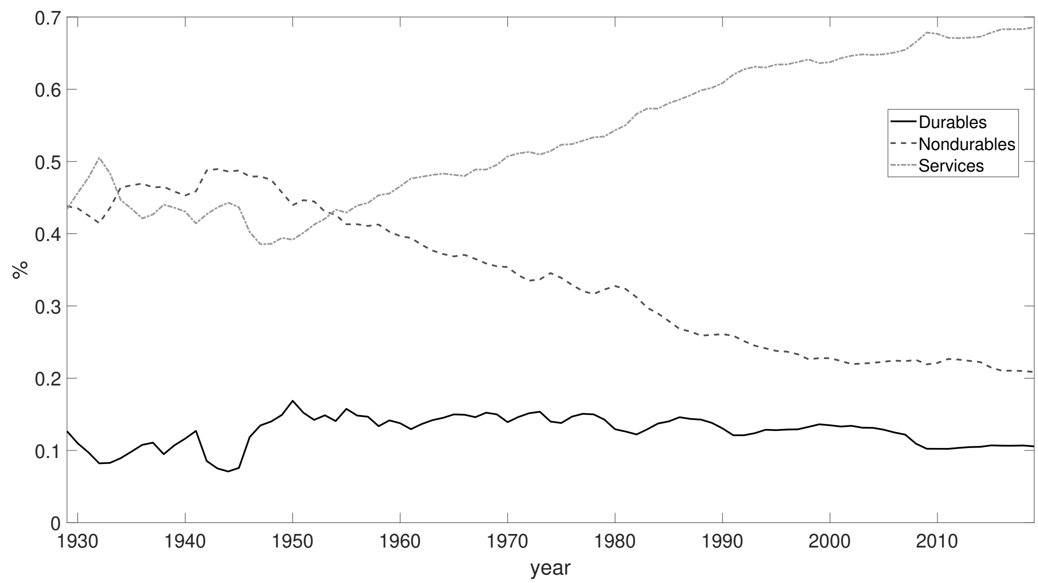
6.1.1 Changes in the Composition of Aggregate Consumption

Historically, there has been a substantial change in the composition of consumption in the US data, with a shift away from an aggregate bundle heavily dependent on nondurable goods to a dominant and much larger share of services. In Figure 2 we show the time series for the consumption shares of the three main components of consumption over time from 1929 to 2019. We can observe that, in more recent years, services account for

[21] Note that since the level of the series is integrated, we cannot consistently estimate parameters $\{\pi_0, \pi_1\}$ with the OLS regression in Table 1. That is another reason why our preferred choice of shock specification isthe one with the ARIMA-BN process as mentioned in the text. This point is also emphasized in Reis (2009).

nearly 70 percent of the bundle while nondurable goods account for 20 percent, a sharp change from the equal split of 45 percent between the two categories prior to the Great Depression.[22]

Figure 2: Consumption shares over time.



Notes: The figure shows the time series for the percent shares of different types of goods in total personal consumption expenditures for the US from 1929 to 2019 (US Bureau of Economic Analysis, 2021c). The solid black line shows the share of durable goods, the dashed dark grey line shows the share of nondurable goods and the dotted-dashed light grey line shows the share of nondurable goods over time.

Given the intrinsically more volatile nature of nondurable goods consumption, which includes, for example, food and energy, such secular change in composition could naturally account for a large part of the decrease in the variance of consumption, potentially undermining the role of stabilization policies. Furthermore, one can observe in the data that the inflection point for the trend in the component shares lies around the end of WWII, coinciding with the point where we estimate the structural break in the variance of the series.

[22] We use the series "Personal Consumption Expenditures by Major Type of Product" from the BEA's NIPA retrievable from Table 2.3.5 (US Bureau of Economic Analysis, 2024).

In order to tackle this potential threat to our identification strategy, we conduct a robustness check on the estimation of σ_u^2, re-running all the regressions in Table 1 but controlling for the shares of nondurable goods and services consumption for the years available in our sample. The estimates can be found in Table A.1 in Appendix B.2. We find that when controlling for the change in consumption composition, the ratio of estimates $\hat{\sigma}_{u,2}^2/\hat{\sigma}_{u,1}^2$ reduces from 0.38 to 0.28 for the transitory shocks specification and increases from 0.14 to 0.16 for the permanent shocks and ARIMA-BN specifications. Hence, in our preferred specification, the ratio of the variance of consumption between the two periods stays relatively the same after using the control shares, ruling out the possibility that such a change would be responsible for a large amount of the decrease in the consumption variance between the two defined periods. Moreover, the small change in the ratio also implies a relatively negligible effect on the value for $\hat{\theta}_2$.

6.2 Time-Varying θ

One critical point in our identification strategy is the sharp division of the whole time series into only two periods, pre- and post-war. More importantly, the period after 1947 exhibits a number of structural changes that fundamentally altered the US economy. Some of these could be the rise of the service industry (as previously discussed) and a sectoral change away from agriculture, skill-biased technological changes, or expansion of the social safety net and the welfare state. All of those could have impacted the income process of the representative household and therefore contributed to the reduction of risk in the consumption process. Such effects could undermine the plausibility of our strong assumption that the majority of the factors behind the substantial decrease in $\hat{\sigma}_{u,2}^2$ could be loaded on θ fixed over time.

In order to tackle this challenge, we relax part of our estimation approach and make use of a methodology that allows us to estimate a time-varying $\hat{\sigma}_{u,t}^2$, and thus a time-varying $\hat{\theta}_t$. First, we focus only on the post-1947 period, which is the part of the sample that is most relevant to capture movements in the structural parameters of the consumption process. As discussed before, it is well-documented by the literature that the period prior to 1947 was one of incipient stabilization policies and we have already been agnostic in our strategy, within the incipient territory, by using a grid for θ_1 in our previous estimation. Then, inspired by Stock and Watson's (2007) analysis of the post-war quarterly inflation process with a parsimonious univariate process, we adapt their methodology to our problem and rewrite our post-war consumption process with a stochastic volatility model. This allows us to work with the desired time-varying variance.

Assume then that the first difference of log-consumption per capita follows a standard stochastic volatility model:

$$\Delta \log(c_t) = \pi_0 + u_t^c, \qquad u_t^c \sim \mathcal{N}(0, e^{h_t}) \tag{31}$$

$$h_t = \mu_h + \phi_h(h_{t-1} - \mu_h) + \varepsilon_t^h, \quad \varepsilon_t^h \sim \mathcal{N}(0, \omega_h^2) \tag{32}$$

where h is the time-varying component of consumption volatility and the process is initialized with $h_1 \sim \mathcal{N}(\mu_h, \omega_h^2/(1-\phi_h^2))$.

We estimate this process with a Bayesian approach using a Markov chain Monte Carlo (MCMC) method developed by Chan and Grant (2016). We use the years 1947-2019 and also multiply the level by 100 for the simulations. To make the computations, we need to start with a choice of suitable priors and degrees of dispersion. Our choices of priors for the parameters are:

$$\begin{aligned} \pi_0 &\sim \mathcal{N}(0, 10) & \mu_h &\sim \mathcal{N}(1, 10) \\ \phi_h &\sim \mathcal{N}(0.9, 0.1^2)\mathbf{1}(|\phi_h| < 1) & \omega_h^2 &\sim \mathcal{IG}(10, 0.36) \end{aligned} \tag{33}$$

where $\mathcal{IG}(\cdot,\cdot)$ denotes an inverse-gamma distribution.

Table 2 shows the posterior means obtained in the Bayesian estimation of the four parameters used in the process. We show the comparison between the prior and posterior distributions obtained in our simulation in Figure A.2 of Appendix B.3. We have chosen informative but initially dispersed priors. The posterior distributions differ from the priors, being tightly centered around the estimated posterior means, indicating that the data are informative for the MCMC sampling process and estimation of the parameters of the stochastic volatility model.

Table 2: Posterior means of the stochastic volatility process.

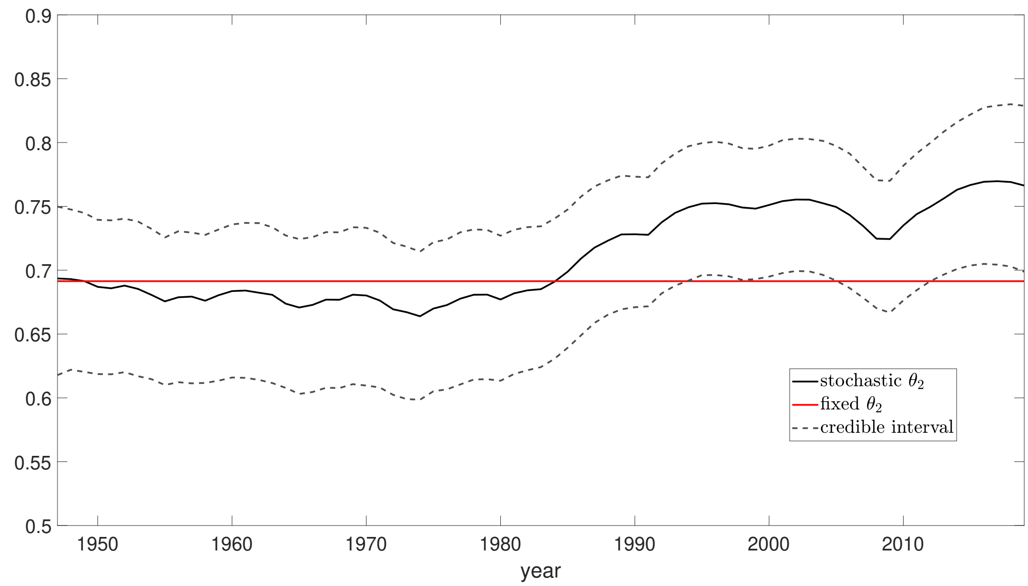
Parameter	Posterior mean	Std. deviation
π_0	1.98	0.20
μ_h	0.96	0.48
ϕ_h	0.88	0.08
ω_h^2	0.04	0.01

Notes: The table shows the posterior means and the standard deviations for the posterior distributions of the four parameters used in the stochastic volatility process described in equations (31) and (32). Estimates were obtained using the method proposed in Chan and Grant (2016).

Given our estimates, we compute the quantiles 16, 50 and 84 of the posterior distribution of the stochastic volatility, i.e., h^{16}, h^{50}, and h^{84}. We use the median, h^{50}, as the main reference point for the implied time-variant $\hat{\theta}_{2,t}$ and the other quantiles to build the credible interval and generate the dispersion bands. Using the same approach as in the previous exercise, the values reported consider $\theta_1 = 0.2$ and $\hat{\sigma}_{u,1} = 0.0021$. Figure 3 shows our results with the time series for the span.[23]

[23] In Appendix B.3, we include Figure A.3 with the estimated time series for the stochastic standard deviation, $\sqrt{e^{h_t^{50}}}$.

Figure 3: Estimated $\hat{\theta}_{2,t}$ using stochastic volatility.



Notes: The figure shows the estimated time series for $\hat{\theta}_{2,t}$. The solid black line shows the values associated with the median quantile of the estimation, while the dashed lines indicating the bands of the credible interval are associated with quantiles 16 and 84. The solid red line shows the $\hat{\theta}_2$ obtained in the estimation shown in Table 1. These values were obtained from the estimation of the stochastic volatility process for the first difference of log-consumption described in equations (31) and (32). The series spans from 1947 through 2019 and is computed considering $\theta_1 = 0.2$ and $\hat{\sigma}_{u,1} = 0.0021$. In Appendix B.3, Figure A.3 shows the estimated time series for the stochastic standard deviation $\sqrt{e^{h_t^{50}}}$.

We can observe that the time series for the $\hat{\theta}_{2,t}$ ranges from around 65 percent to 77 percent. These values do not lie far from our original estimated value, 69 percent, with the two-period identification methodology. For a large part of the time series, namely, from the end of WWII until the late 80s, the values for the span remained almost flat, gravitating around the center value of 69 percent. Starting in the early 90s and beyond, the value for time-varying θ starts to climb, potentially settling at a new, higher level, around 75 percent, with a few oscillations around 2010.

These results and the behavior of the series tell us that our initial strategy, along with the estimate we obtained with it, are not far from the ones obtained with a more complex and flexible fitting of the data and can then suffice for being used in our welfare cost calculations. This is not only reassuring but convenient, as our proposed methodology can accommodate only one span at a time. Second, the climb in the value of $\hat{\theta}_2$ starting in the early 1990s until today is consistent with the overall compression of the estimated consumption volatility in this period shown in Figure 2b, which also exhibits an increased persistence, a factor that might be attributed to the "Great Moderation" (Stock and Watson, 2002). This further highlights the existence of ongoing stabilization policies that are reflected in a less volatile consumption process, exactly the effect that the implied θ aims to capture.

7 Welfare Costs of Economic Fluctuations

With the estimated values for $\hat{\theta}_2$ and $\hat{\sigma}_\varepsilon^2$, we can now turn back to the calculation of our decomposition for λ^T, λ^B, and λ^R shown in Theorem 1. We show all our results in Table 3. The numbers are obtained by plugging the estimates in Table 1 into equations (10) through (20) and are shown for all the implied $\hat{\theta}_2$ from our grid for θ_1 and for four different values of the degree of relative risk aversion, γ. For the permanent and ARIMA-BN shocks, the values reported consider $\beta = 0.96$.[24] We also provide a measure that is more naturally comparable to the ones shown in the literature that computes costs with the absence of the span θ, which is represented by λ^{lit} placed in the last column of the table. The derivation of this equivalent cost is straightforward and hence we leave it to Appendix A.4.

[24] In Appendix C.1, we report the results for $\beta \in \{0.95, 0.96, 0.97\}$.

Table 3: Decomposition of the welfare cost of total economic fluctuations.

	λ^T				λ^B				λ^R				λ^{lit}
Transitory shocks													
$\hat{\theta}_2$	0.37	0.44	0.50	0.56	0.37	0.44	0.50	0.56	0.37	0.44	0.50	0.56	-
$\gamma = 1$	0.32	0.40	0.51	0.66	0.12	0.17	0.25	0.37	0.20	0.23	0.25	0.29	0.13
$\gamma = 2.5$	0.81	1.00	1.27	1.66	0.42	0.58	0.82	1.17	0.39	0.42	0.44	0.48	0.32
$\gamma = 5$	1.63	2.01	2.55	3.35	0.91	1.27	1.78	2.53	0.71	0.74	0.76	0.80	0.64
$\gamma = 7.5$	2.45	3.04	3.86	5.07	1.40	1.96	2.74	3.90	1.03	1.06	1.08	1.12	0.96
Permanent shocks													
$\hat{\theta}_2$	0.63	0.67	0.70	0.74	0.63	0.67	0.70	0.74	0.63	0.67	0.70	0.74	-
$\gamma = 1$	2.63	3.92	4.99	6.65	1.64	3.02	4.11	5.74	0.97	0.88	0.84	0.86	0.36
$\gamma = 2.5$	3.25	4.89	6.33	8.91	2.15	3.92	5.40	7.97	1.08	0.94	0.88	0.88	0.52
$\gamma = 5$	4.14	6.28	8.34	13.01	2.89	5.21	7.34	11.99	1.21	1.02	0.93	0.90	0.63
$\gamma = 7.5$	5.44	8.39	11.60	15.47	4.00	7.19	10.52	14.39	1.39	1.12	0.98	0.95	0.69
ARIMA-BN													
$\hat{\theta}_2$	0.61	0.65	0.69	0.73	0.61	0.65	0.69	0.73	0.61	0.65	0.69	0.73	-
$\gamma = 1$	5.15	6.40	8.16	10.79	3.13	4.13	5.58	7.77	1.96	2.18	2.45	2.81	0.75
$\gamma = 2.5$	6.75	8.49	11.06	15.08	5.13	6.73	9.11	12.86	1.54	1.65	1.79	1.96	0.94
$\gamma = 5$	8.16	10.63	14.65	22.16	6.71	9.09	12.96	20.26	1.36	1.41	1.49	1.58	1.03
$\gamma = 7.5$	9.64	13.37	20.92	51.68	8.25	11.88	19.28	49.52	1.29	1.33	1.38	1.44	1.06

Notes: The table displays the computed parameters for the decomposition of the welfare cost of total economic fluctuations. The numbers are obtained using equations (10) through (20) with the estimates shown in Table 1. All of the entries are in percentages of lifetime consumption. We also report an extra welfare cost measure, λ^{lit}, described in Appendix A.4. We report numbers for the relative degree of risk aversion $\gamma \in \{1, 2.5, 5, 7.5\}$, for the implied $\hat{\theta}_2$ along the grid for $\theta_1 \in \{0, 0.1, 0.2, 0.3\}$, and with a calibrated $\beta = 0.96$ for the permanent shocks and ARIMA-BN process.

We focus on the usual level of relative risk aversion used in the literature, $\gamma = 2.5$. For transitory shocks with a span of $\hat{\theta}_2 = 0.50$, the total cost, λ^T, is 1.27 percent of lifetime consumption, being divided into 0.82 stemming from the benefit of current stabilization policies and 0.44 of residual cost. In this case, we recover modest numbers for the costs, though 1.27 percent of lifetime consumption is already at the higher levels estimated in the literature. Nonetheless, the comparison with λ^{lit} implies a small difference of 12 percentage points with λ^R, showing the limitations imposed by the original shock structure.

With permanent shocks we obtain an overall increase of all λ's with a substantially high $\lambda^T = 6.33$, with more than 85 percent of this stemming from λ^B, hence already being stabilized. We end up with 0.88 percent of consumption still left to be smoothed by policies. If we compare this result to the equivalent calculation obtained in the literature, λ^{lit}, we can see that, for our permanent shocks specification, our residual cost is almost 70 percent larger than what a measure with the absence of θ would imply.

Finally, we can focus on our preferred specification of the shocks, the ARIMA-BN process structure, which is the one that better fits the time-series characteristics of the log-consumption data. Results shown in the bottom third of Table 3 point to the high welfare costs of total economic fluctuations. The total cost, λ^T, is 11 percent of lifetime consumption with $\lambda^B = 9.11$, or 82 percent of it represented as the benefit of ongoing policies. This leaves us with a residual of $\lambda^R = 1.79$ yet to be smoothed, almost double the value of λ^{lit}.

More importantly, beyond finding high levels of costs for λ^T, the approach is able to unveil how much of the total welfare costs is left unaccounted for if one focuses only on the residual measures. Fixing $\gamma = 2.5$, even if we assume a zero effect of stabilization policies in the pre-1947 period, there would still be 5.13 percent of lifetime consumption accruing to the benefit of ongoing policies. Had we assumed a $\theta_1 = 0.3$, the highest value in our grid, we would then jump to almost 13 percent of lifetime consumption smoothed by the stabilization policies that are already set in place. If we return to $\hat{\theta}_2 = 0.69$ and let $\gamma = 5$, we have that the total cost is 14.65 percent of lifetime consumption, out of which 88 percent is already being stabilized.

Overall, the numbers we find for the total costs at our preferred specification lie at a higher value than the figures often estimated in the literature, especially when considering the cases with standard CRRA preferences. Apart from the comparison with λ^{lit} within our own framework, we can also simply benchmark the values against some of the numbers estimated with other methodologies. Barlevy (2005) provides an excellent summary of the early literature in which costs vary from 0.01 to 8.0 percent. One of the exceptions is Dolmas (1998), who finds values for the costs above 20 percent when using first-order risk aversion, permanent shocks, and small elasticities of substitution. Closer to our approach and with a focus on the statistical properties of the consumption growth series, Reis (2009) finds values that range from 0.5 to 5 percent, with higher numbers - but smaller than 8 percent - when considering ARMA processes coupled with high risk-aversion. Also, with a similar approach, Guillén et al. (2014) find at most 5 percent, even when including the pre-WWII high-volatility years in the estimation and high risk aversion in an environment blending permanent and transitory shocks.

The results also allow us to explore a simple theoretical aspect that allows us to understand how the concave utility interacts with our proposed decomposition and the parameter θ. If we fix a given level of the measured span of policies, the marginal benefit of smoothing the residual fluctuations in proportion to the total welfare cost, i.e., λ^R/λ^T, is decreasing in the relative degree of risk aversion. Risk-averse consumers tend to value relatively more the benefit generated by the ongoing stabilization policies, going up as much as 92 percent of the total welfare cost with the ARIMA-BN process when γ is at the highest level considered.

8 Robustness - Structural Break

An important potential concern is the presence of structural breaks in a long time series. We have already identified a structural break in the volatility of our consumption series, but it is also important to test whether we find any in the historical path of the consumption data. We apply the methodology developed in Bai and Perron (1998, 2003) to test structural breaks in our sample. For the transitory shock version we test a structural break in log-consumption and we find breaks in 1879, 1931, and 1993. We also test a structural break in the first difference of log-consumption, which is the series used in our main analysis that accommodates permanent and persistent shocks. We obtain a scaled F-statistic of 9.31 (with a critical value of 8.58), indicating a break in 1934.[25]

[25] In our tests, we allow for at most 5 breaks in the time series. The tests indicate only one break in the first difference of log-consumption (1934) and indicate 3 breaks in the log-consumption (1879, 1931, and 1993). For 1931 the scaled F-statistic is 1221.88 with a critical value of 11.47.

We use these breaks to create a new division of the sub-samples that are used in our identification strategy. Since the level and the first difference both indicate a break in the 1930s, this is then the most relevant period for adjustment of the data. The first sub-sample we define considers the years from the beginning of the sample until the years of the structural breaks, and a second sub-sample, as in the main text, considers the years after WWII. To keep the sub-samples the same size for our estimations with all types of shocks, we set the first sub-sample for the years between 1835 and 1930.

The results of the estimated parameters along with the implied $\hat{\theta}_2$ are presented in Table 4. If we compare those results with the results in Table 1 in our main exercise, we note that the estimations imply only a marginal change in the implied parameters with all three shock structures. In Appendix C.2, Tables A.4 and A.5 present the welfare cost calculations for our λ's using the implied parameters in Table 4. For our preferred parameters, we find a difference of only 1 percentage point for the estimate of $\hat{\theta}_2$ and the computed λ^T, both with a lower value.

Table 4: Structural break - Estimation

Transitory shocks				Permanent shocks				ARIMA-BN			
Estimated parameters				Estimated parameters				Estimated parameters			
	$\hat{\pi}_0$	$\hat{\pi}_1$	$\hat{\sigma}_u^2$		$\hat{\pi}_0$	$\hat{\pi}_1$	$\hat{\sigma}_u^2$		$\hat{\pi}_0$	$\hat{\phi}_1$	$\hat{\sigma}_u^2$
1835 - 1930	7.726	0.0114	0.0055	1835 - 1930	0.0096	-	0.0019	1835 - 1930	0.0096	-	0.0019
	(.0155)	(.0003)			(.0045)				(.0045)		
1947 - 2019	6.6156	0.0219	0.0025	1947 - 2019	0.0203	-	0.0003	1947 - 2019	0.0189	0.2919	0.0003
	(.0427)	(.0003)			(.0020)				(.0030)	(.1321)	
Implied parameters				Implied parameters				Implied parameters			
θ_1	$\hat{\theta}_2$	$\hat{\alpha}_1$	$\hat{\sigma}_\varepsilon^2$	θ_1	$\hat{\theta}_2$	$\hat{\alpha}_1$	$\hat{\sigma}_\varepsilon^2$	θ_1	$\hat{\theta}_2$	$\hat{\alpha}_1$	$\hat{\sigma}_\varepsilon^2$
0.00	0.3215	0.0222	0.0055	0.00	0.6158	0.0208	0.0019	0.00	0.6011	0.0277	0.0019
0.10	0.3893	0.0222	0.0068	0.10	0.6542	0.0209	0.0024	0.10	0.6410	0.0277	0.0024
0.20	0.4572	0.0222	0.0086	0.20	0.6926	0.0209	0.0030	0.20	0.6809	0.0278	0.0030
0.30	0.5250	0.0222	0.0113	0.30	0.7311	0.0210	0.0040	0.30	0.7208	0.0279	0.0040

Notes: The table displays the estimated parameters obtained by running regression (22) for transitory shocks and regression (25) for permanent and ARIMA-BN shocks. The time series used is our sample of the augmented Barro and Ursúa (2010) data with sub-periods 1835-1930 and 1947-2019. The series is I(1) as identified by the ADF, PP, and DF-GLS tests. The first difference of the series is identified, by both the AIC and BIC criteria, as an ARMA (0,0) for the pre-1930 period and an ARMA(1,0) for the subsequent years. The implied parameters are obtained using the formulas described in the text and equation (30).

9 Conclusion

In this paper we revisited the long-standing issue of the welfare costs of business cycles with a focus on unveiling the extent to which ongoing stabilization policies are smoothing observed consumption. We rooted our approach in the novel modelling that all data we gather on consumption are subject to the policy status quo and we provided a decomposition for macroeconomic fluctuations. We recovered the total welfare costs of business cycles by disentangling them into the benefit of current policies and the residual yet to be flattened.

We also conducted an empirical analysis with the goal of identifying our key decomposition parameter, the span of stabilization power, from the historical consumption data. In doing so, we profited from the observation that there is a discontinuous decrease in the series' volatility after WWII, a fact widely documented by a vast literature in macroeconomics. With the proper strategy, we were able to recover estimates from the data and found that the span of stabilization power, in our preferred shock structure and parameter space, is approximately 69 percent and the welfare costs of total economic fluctuations are around 11 percent of permanent consumption, with 9 percent of it already being smoothed by ongoing policies and 1.8 percent left as a residual.

Our paper abstracts from some key aspects that are relevant to our question, such as different types of consumption goods, heterogeneity, and distributional aspects that shed a stronger light on consumption and risk inequality. We also take a simplified view of the role of stabilization policies and technological changes in the post-war US economy. We attempted to tackle part of the latter issue by constructing a time-varying span of stabilization power that yields estimates similar to those in our original analysis. However, we understand that they are all critical considerations that are worth a detailed exploration that could potentially expand our analysis. But for the moment, we leave them for future research.

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Appendix

"The Welfare Costs of Business Cycles Unveiled: Measuring the Extent of Stabilization Policies"

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A Proofs

Below we outline the proofs for Lemmas 1, 2, 3, Propositions 1, 2, 3 and for Theorem 1.

Lemma 1. *Under Assumption 1 and CRRA utility* (5),

$$\sum_{t=0}^{\infty} \beta^t u\left(\bar{C}_t\right) = \begin{cases} \frac{\ln \alpha_0}{1-\beta} + \frac{\beta \ln(1+\alpha_1)}{(1-\beta)^2}, & \text{if } \gamma = 1 \\ \tilde{\alpha}_0 \sum_{t=0}^{\infty} \Gamma^t, & \text{if } \gamma > 1 \end{cases} \tag{A.1}$$

where $\tilde{\alpha}_0 \equiv (1-\gamma)^{-1}\alpha_0^{1-\gamma}$.

Proof of Lemma 1. Consider a $\gamma = 1$. Then,

$$\sum_{t=0}^{\infty} \beta^t \ln\left(\bar{C}_t\right) = \sum_{t=0}^{\infty} \beta^t \left(\ln \alpha_0 + t \ln\left(1+\alpha_1\right)\right) = \frac{\ln \alpha_0}{1-\beta} + \frac{\beta \ln\left(1+\alpha_1\right)}{(1-\beta)^2}. \tag{A.2}$$

When $\gamma > 1$,

$$\sum_{t=0}^{\infty} \beta^t (1-\gamma)^{-1} \left(\bar{C}_t\right)^{1-\gamma} = (1-\gamma)^{-1}\alpha_0^{1-\gamma} \sum_{t=0}^{\infty} \left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t = \tilde{\alpha}_0 \sum_{t=0}^{\infty} \Gamma^t. \tag{A.3}$$

■

Lemma 2. *Consider an arbitrary constant* $k > 0$*. Under Assumption 1 and CRRA utility* (5),

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty} \beta^t u\left((1+k)\tilde{C}_t\right)\right] = \begin{cases} \frac{\ln(1+k)}{1-\beta} + \frac{\ln \alpha_0}{1-\beta} + \frac{\beta \ln(1+\alpha_1)}{(1-\beta)^2} - \frac{1}{2}\sum_{t=0}^{\infty} \beta^t \sigma_t^2, & \text{if } \gamma = 1 \\ \tilde{\alpha}_0 (1+k)^{1-\gamma} \sum_{t=0}^{\infty} \Gamma^t \exp\left\{-0.5\gamma\left(1-\gamma\right)\sigma_t^2\right\}, & \text{if } \gamma > 1 \end{cases} \tag{A.4}$$

where $\tilde{\alpha}_0 \equiv (1-\gamma)^{-1}\alpha_0^{1-\gamma}$.

Proof of Lemma 2. For the case where $\gamma = 1$,

$$\begin{aligned}
\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t \ln\left[(1+k)\tilde{C}_t\right]\right] &= E_0\left[\sum_{t=0}^{\infty}\beta^t\left(\ln(1+k)+\ln\alpha_0+t\ln\left(1+\alpha_1\right)+x_t-\frac{1}{2}\sigma_t^2\right)\right] \\
&= \sum_{t=0}^{\infty}\beta^t\left(\ln(1+k)+\ln\alpha_0+t\ln\left(1+\alpha_1\right)+E_0\left[x_t\right]-\frac{1}{2}\sigma_t^2\right) \\
&= \frac{\ln(1+k)}{1-\beta}+\frac{\ln\alpha_0}{1-\beta}+\frac{\beta\ln\left(1+\alpha_1\right)}{(1-\beta)^2}-\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\sigma_t^2,
\end{aligned}$$

using the fact that $E_0\left[x_t\right]=0$.

For the case where $\gamma > 1$,

$$\begin{aligned}
\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\frac{\left[(1+k)\tilde{C}_t\right]^{1-\gamma}}{1-\gamma}\right] &= (1-\gamma)^{-1}\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left[(1+k)\alpha_0\left(1+\alpha_1\right)^t\exp\left\{x_t-0.5\sigma_t^2\right\}\right]^{1-\gamma}\right] \\
&= \tilde{\alpha}_0(1+k)^{1-\gamma}\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t\times\ldots \\
&\quad \ldots\exp\left\{-0.5\left(1-\gamma\right)\sigma_t^2\right\}E_0\left[\exp\left\{\left(1-\gamma\right)x_t\right\}\right].
\end{aligned}$$

Note that

$$\mathbb{E}_0\left[\exp\left\{\left(1-\gamma\right)x_t\right\}\right]=\exp\left\{E_0\left[\left(1-\gamma\right)x_t\right]+0.5Var_0\left[\left(1-\gamma\right)x_t\right]\right\}=\exp\left\{0.5\left(1-\gamma\right)^2\sigma_t^2\right\}.$$

Thus,

$$\begin{aligned}
\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\frac{\left[(1+k)\tilde{C}_t\right]^{1-\gamma}}{1-\gamma}\right] &= \tilde{\alpha}_0(1+k)^{1-\gamma}\sum_{t=0}^{\infty}\Gamma^t\exp\left\{-0.5\left(1-\gamma\right)\sigma_t^2\right\}\exp\left\{0.5\left(1-\gamma\right)^2\sigma_t^2\right\} \\
&= \tilde{\alpha}_0(1+k)^{1-\gamma}\sum_{t=0}^{\infty}\Gamma^t\exp\left\{-0.5\gamma\left(1-\gamma\right)\sigma_t^2\right\}.
\end{aligned}$$

■

Lemma 3. *Consider an arbitrary constant* $\ell > 0$. *Under Assumption 1 and CRRA utility (5),*

$$\sum_{t=0}^{\infty}\beta^t u\left((1+\ell)C_t(\theta)\right)=$$

$$\begin{cases} \frac{\ln(1+\ell)}{1-\beta} + \frac{\ln\alpha_0}{1-\beta} + \frac{\beta\ln(1+\alpha_1)}{(1-\beta)^2} - \frac{1}{2}(1-\theta)\sum_{t=0}^{\infty}\beta^t\sigma_t^2, & \textit{if } \gamma = 1 \\ \tilde{\alpha}_0(1+\ell)^{1-\gamma}\sum_{t=0}^{\infty}\Gamma^t \exp\left\{-0.5(1-\gamma)(1-\theta)(\theta+\gamma-\gamma\theta)\sigma_t^2\right\}, & \textit{if } \gamma > 1 \end{cases} \tag{A.5}$$

where $\tilde{\alpha}_0 \equiv (1-\gamma)^{-1}\alpha_0^{1-\gamma}$.

Proof of Lemma 3. Again, when $\gamma = 1$,

$$\begin{aligned} \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t \ln\left[(1+\ell)C_t(\theta)\right]\right] &= \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t \ln\left[(1+\ell)\alpha_0(1+\alpha_1)^t \exp\left\{(1-\theta)\left[x_t - 0.5\sigma_t^2\right]\right\}\right]\right] \\ &= \sum_{t=0}^{\infty}\beta^t\left(\ln(1+\ell) + \ln\alpha_0 + t\ln(1+\alpha_1) + (1-\theta)\left[E_0[x_t] - 0.5\sigma_t^2\right]\right) \\ &= \frac{\ln(1+\ell)}{1-\beta} + \frac{\ln\alpha_0}{1-\beta} + \frac{\beta\ln(1+\alpha_1)}{(1-\beta)^2} - \frac{1-\theta}{2}\sum_{t=0}^{\infty}\beta^t\sigma_t^2, \end{aligned}$$

given that $E_0[x_t] = 0$. With $\gamma > 1$,

$$\begin{aligned} \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t \frac{\left[(1+\ell)C_t(\theta)\right]^{1-\gamma}}{1-\gamma}\right] &= (1-\gamma)^{-1}\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left[(1+\ell)C_t(\theta)\right]^{1-\gamma}\right] \\ &= (1-\gamma)^{-1}\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left[(1+\ell)\alpha_0(1+\alpha_1)^t + \ldots\right.\right. \\ &\ldots \left.\left.\exp\left\{(1-\theta)\left[x_t - 0.5\sigma_t^2\right]\right\}\right]^{1-\gamma}\right] \\ &= \tilde{\alpha}_0(1+\ell)^{1-\gamma}\sum_{t=0}^{\infty}\left[\beta(1+\alpha_1)^{1-\gamma}\right]^t \times \ldots \\ &\ldots \exp\left\{-0.5(1-\theta)(1-\gamma)\sigma_t^2\right\} E_0\left[\exp\left\{(1-\theta)(1-\gamma)x_t\right\}\right]. \end{aligned}$$

Note that

$$\begin{aligned} \mathbb{E}_0\left[\exp\left\{(1-\theta)(1-\gamma)x_t\right\}\right] &= \exp\left\{\mathbb{E}_0\left[(1-\theta)(1-\gamma)x_t\right] + 0.5Var_0\left[(1-\theta)(1-\gamma)x_t\right]\right\} \\ &= \exp\left\{0.5(1-\theta)^2(1-\gamma)^2\sigma_t^2\right\}. \end{aligned}$$

And,

$$\begin{aligned}
& \exp\left\{-0.5\left(1-\theta\right)\left(1-\gamma\right)\sigma_t^2\right\}\mathbb{E}_0\left[\exp\left\{\left(1-\theta\right)\left(1-\gamma\right)x_t\right\}\right] \\
= & \exp\left\{-0.5\left(1-\theta\right)\left(1-\gamma\right)\sigma_t^2\right\}\exp\left\{0.5\left(1-\theta\right)^2\left(1-\gamma\right)^2\sigma_t^2\right\} \\
= & \exp\left\{-0.5\left(1-\theta\right)\left(1-\gamma\right)\left(\gamma+\theta-\gamma\theta\right)\sigma_t^2\right\}.
\end{aligned}$$

Thus,

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\frac{\left[(1+\ell)C_t\left(\theta\right)\right]^{1-\gamma}}{1-\gamma}\right] = \tilde{\alpha}_0(1+\ell)^{1-\gamma}\sum_{t=0}^{\infty}\Gamma^t\exp\left\{-0.5\left(1-\theta\right)\left(1-\gamma\right)\left(\gamma+\theta-\gamma\theta\right)\sigma_t^2\right\}$$

■

Proof of Proposition 1. Replace k with λ^B in Lemma 2, use $\ell = 0$ in Lemma 3, and then solve equation (3) for λ^B. The assumptions guarantee that $\lambda^B < \infty$. ■

Proof of Proposition 2. We use $\ell = \lambda^R$ in Lemma 3 and the results in Lemma 1 for solving equation (4) for λ^R. The assumptions guarantee that $\lambda^R < \infty$. ■

Proof of Proposition 3. We use $k = \lambda^T$ in Lemma 2 and Lemma 1 in equation (1). Then, we solve it for λ^T. The assumptions guarantee that $\lambda^T < \infty$. ■

Proof of Theorem 1. For $\gamma = 1$, we have

$$\begin{aligned}
\left(1+\lambda^B\right)\left(1+\lambda^R\right) &= \exp\left\{\theta\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\sigma_t^2\right\}\exp\left\{\left(1-\theta\right)\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\sigma_t^2\right\} \\
&= \exp\left\{\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\sigma_t^2\right\} = 1+\lambda^T
\end{aligned}$$

Now, for $\gamma > 1$, we have

$$\begin{aligned}\left(1+\lambda^{B}\right)^{1-\gamma}\left(1+\lambda^{R}\right)^{1-\gamma} &= \frac{\sum_{t=0}^{\infty}\Gamma^{t}e^{-0.5(1-\gamma)(1-\theta)(\theta+\gamma-\gamma\theta)\sigma_t^2}}{\sum_{t=0}^{\infty}\Gamma^{t}e^{-0.5\gamma(1-\gamma)\sigma_t^2}}\frac{\sum_{t=0}^{\infty}\Gamma^{t}}{\sum_{t=0}^{\infty}\Gamma^{t}e^{-0.5(1-\theta)(1-\gamma)(\gamma+\theta-\gamma\theta)\sigma_t^2}}\\ \Longleftrightarrow \left(1+\lambda^{B}\right)\left(1+\lambda^{R}\right) &= \left[\frac{\sum_{t=0}^{\infty}\Gamma^{t}}{\sum_{t=0}^{\infty}\Gamma^{t}e^{-0.5\gamma(1-\gamma)\sigma_t^2}}\right]^{\frac{1}{1-\gamma}} = 1+\lambda^{T}\end{aligned}$$

■

A.1 Calculations for the Applications

A.1.1 Example 1 (Lucas, 1987):

For $\gamma = 1$:

$$\lambda^{T} = \exp\left\{\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^{t}\sigma_t^2\right\} - 1 = \exp\left\{\frac{1}{2}\sigma_\varepsilon^2\right\} - 1 \tag{A.6}$$

$$\lambda^{B} = \exp\left\{\theta\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^{t}\sigma_t^2\right\} - 1 = \exp\left\{\theta\frac{1-\beta}{2}\sigma_\varepsilon^2\frac{1}{1-\beta}\right\} - 1 = \exp\left\{\frac{\theta}{2}\sigma_\varepsilon^2\right\} - 1 \tag{A.7}$$

$$\lambda^{R} = \exp\left\{(1-\theta)\frac{1-\beta}{2}\sigma_\varepsilon^2\frac{1}{1-\beta}\right\} - 1 = \exp\left\{\frac{1-\theta}{2}\sigma_\varepsilon^2\right\} - 1 \tag{A.8}$$

For $\gamma > 1$:

$$\begin{aligned}\lambda^{T} &= \left[\frac{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^{t}}{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^{t}\exp\left\{0.5\gamma\left(\gamma-1\right)\sigma_t^2\right\}}\right]^{\frac{1}{1-\gamma}} - 1\\ &= \left[\frac{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^{t}}{\exp\left\{0.5\gamma\left(\gamma-1\right)\sigma_\varepsilon^2\right\}\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^{t}}\right]^{\frac{1}{1-\gamma}} - 1\\ &= \left[\frac{1}{\exp\left\{0.5\gamma\left(\gamma-1\right)\sigma_\varepsilon^2\right\}}\right]^{\frac{1}{1-\gamma}} - 1 = \exp\left\{\frac{1}{2}\gamma\sigma_\varepsilon^2\right\} - 1 \end{aligned} \tag{A.9}$$

$$\lambda^B = \left[\frac{\exp\left\{-0.5\left(1-\theta\right)\left(1-\gamma\right)\left[\gamma+\theta-\theta\gamma\right]\sigma^2\right\}\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t}{\exp\left\{0.5\gamma\left(\gamma-1\right)\sigma_\varepsilon^2\right\}\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t} \right]^{\frac{1}{1-\gamma}} - 1$$

$$= \left[\frac{\exp\left\{-0.5\left(1-\theta\right)\left(1-\gamma\right)\left[\gamma+\theta-\theta\gamma\right]\sigma_\varepsilon^2\right\}}{\exp\left\{-0.5\gamma\left(1-\gamma\right)\sigma_\varepsilon^2\right\}} \right]^{\frac{1}{1-\gamma}} - 1$$

$$= \exp\left\{\frac{1}{2}\left[\gamma-\left(1-\theta\right)\left(\gamma+\theta-\theta\gamma\right)\right]\sigma_\varepsilon^2\right\} \tag{A.10}$$

$$\lambda^R = \left[\frac{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t}{\exp\left\{-0.5\left(1-\theta\right)\left(1-\gamma\right)\left[\gamma+\theta-\theta\gamma\right]\sigma_\varepsilon^2\right\}\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t} \right]^{\frac{1}{1-\gamma}} - 1$$

$$= \left[\frac{1}{\exp\left\{-0.5\left(1-\theta\right)\left(1-\gamma\right)\left[\gamma+\theta-\theta\gamma\right]\sigma_\varepsilon^2\right\}} \right]^{\frac{1}{1-\gamma}} - 1$$

$$= \exp\left\{\frac{1}{2}\left(1-\theta\right)\left(\gamma+\theta-\theta\gamma\right)\sigma_\varepsilon^2\right\} \tag{A.11}$$

A.2 Example 2 (Obstfeld, 1994):

For $\gamma = 1$:

$$\lambda^T = \exp\left\{\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\sigma_t^2\right\} - 1 = \exp\left\{\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\left(t\sigma_\varepsilon^2+\sigma_\varepsilon^2\right)\right\} - 1$$

$$= \exp\left\{\frac{1-\beta}{2}\left[\frac{\beta}{\left(1-\beta\right)^2}+\frac{1}{1-\beta}\right]\sigma_\varepsilon^2\right\} - 1 = \exp\left\{\frac{1}{2}\frac{1}{1-\beta}\sigma_\varepsilon^2\right\} - 1 \tag{A.12}$$

$$\lambda^B = \exp\left\{\theta\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\sigma_t^2\right\} - 1 = \exp\left\{\theta\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\left(t\sigma_\varepsilon^2+\sigma_\varepsilon^2\right)\right\} - 1$$

$$= \exp\left\{\frac{\theta}{2}\left[\frac{\beta+1-\beta}{1-\beta}\right]\sigma_\varepsilon^2\right\} - 1 = \exp\left\{\frac{\theta}{2}\frac{1}{1-\beta}\sigma_\varepsilon^2\right\} - 1 \tag{A.13}$$

$$
\begin{aligned}
\lambda^R &= \exp\left\{(1-\theta)\,\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\sigma_t^2\right\}-1=\exp\left\{(1-\theta)\,\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\left(t\sigma_\varepsilon^2+\sigma_\varepsilon^2\right)\right\}-1\\
&= \exp\left\{\frac{1-\theta}{2}\left[\frac{\beta+1-\beta}{1-\beta}\right]\sigma_\varepsilon^2\right\}-1=\exp\left\{\frac{1-\theta}{2}\frac{1}{1-\beta}\sigma_\varepsilon^2\right\}-1 \qquad \text{(A.14)}
\end{aligned}
$$

For $\gamma > 1$:

$$
\begin{aligned}
\lambda^T &= \left[\frac{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t}{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t\exp\left\{0.5\gamma\left(\gamma-1\right)\left(t\sigma_\varepsilon^2+\sigma_\varepsilon^2\right)\right\}}\right]^{\frac{1}{1-\gamma}}-1\\
&= \left[\frac{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t}{\exp\left\{0.5\gamma\left(\gamma-1\right)\sigma_\varepsilon^2\right\}\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\exp\left\{0.5\gamma\left(\gamma-1\right)\sigma_\varepsilon^2\right\}\right]^t}\right]^{\frac{1}{1-\gamma}}-1\\
&= \left[\frac{1}{1-\beta\left(1+\alpha_1\right)^{1-\gamma}}\frac{1-\beta\left(1+\alpha_1\right)^{1-\gamma}\exp\left\{0.5\gamma\left(\gamma-1\right)\sigma_\varepsilon^2\right\}}{\exp\left\{0.5\gamma\left(\gamma-1\right)\sigma_\varepsilon^2\right\}}\right]^{\frac{1}{1-\gamma}}-1\\
&= \exp\left\{0.5\gamma\sigma_\varepsilon^2\right\}\left[\frac{1-\beta\left(1+\alpha_1\right)^{1-\gamma}\exp\left\{0.5\gamma\left(\gamma-1\right)\sigma_\varepsilon^2\right\}}{1-\beta\left(1+\alpha_1\right)^{1-\gamma}}\right]^{\frac{1}{1-\gamma}}-1 \qquad \text{(A.15)}
\end{aligned}
$$

$$
\begin{aligned}
&\lambda^B =\\
&= \left[\frac{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t\exp\left\{-0.5\left(1-\theta\right)\left(1-\gamma\right)\left[\gamma+\theta-\theta\gamma\right]\left(t\sigma_\varepsilon^2+\sigma_\varepsilon^2\right)\right\}}{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t\exp\left\{0.5\gamma\left(\gamma-1\right)\left(t\sigma_\varepsilon^2+\sigma_\varepsilon^2\right)\right\}}\right]^{\frac{1}{1-\gamma}}-1\\
&= \left[\frac{\exp\left\{-0.5\left(1-\theta\right)\left(1-\gamma\right)\left[\gamma+\theta-\theta\gamma\right]\sigma_\varepsilon^2\right\}}{\exp\left\{-0.5\gamma\left(1-\gamma\right)\sigma_\varepsilon^2\right\}}\frac{\frac{1}{1-\beta(1+\alpha_1)^{1-\gamma}\exp\left\{-0.5(1-\theta)(1-\gamma)[\gamma+\theta-\theta\gamma]\sigma_\varepsilon^2\right\}}}{\frac{1}{1-\beta(1+\alpha_1)^{1-\gamma}\exp\left\{0.5\gamma(\gamma-1)\sigma_\varepsilon^2\right\}}}\right]^{\frac{1}{1-\gamma}}-1\\
&= \frac{\exp\left\{0.5\gamma\sigma^2\right\}}{\exp\left\{0.5\left(1-\theta\right)\left[\gamma+\theta-\theta\gamma\right]\sigma^2\right\}}\\
&\quad\times\left[\frac{1-\beta\left(1+\alpha_1\right)^{1-\gamma}\exp\left\{0.5\gamma\left(\gamma-1\right)\sigma_\varepsilon^2\right\}}{1-\beta\left(1+\alpha_1\right)^{1-\gamma}\exp\left\{-0.5\left(1-\theta\right)\left(1-\gamma\right)\left[\gamma+\theta-\theta\gamma\right]\sigma_\varepsilon^2\right\}}\right]^{\frac{1}{1-\gamma}}-1 \qquad \text{(A.16)}
\end{aligned}
$$

$$\lambda^R = \left[\frac{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t}{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t \exp\left\{-0.5\left(1-\theta\right)\left(1-\gamma\right)\left[\gamma+\theta-\theta\gamma\right]\left(t\sigma_\varepsilon^2+\sigma_\varepsilon^2\right)\right\}}\right]^{\frac{1}{1-\gamma}} - 1$$
$$= \left[\frac{1}{\exp\left\{-0.5\left(1-\theta\right)\left(1-\gamma\right)\left[\gamma+\theta-\theta\gamma\right]\sigma_\varepsilon^2\right\}}\right]^{\frac{1}{1-\gamma}}$$
$$\times \left[\frac{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t}{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\exp\left\{-0.5\left(1-\theta\right)\left(1-\gamma\right)\left[\gamma+\theta-\theta\gamma\right]\sigma_\varepsilon^2\right\}\right]^t}\right]^{\frac{1}{1-\gamma}} - 1$$
$$= \frac{1}{\exp\left\{-0.5\left(1-\theta\right)\left[\gamma+\theta-\theta\gamma\right]\sigma_\varepsilon^2\right\}}$$
$$\times \left[\frac{1-\beta\left(1+\alpha_1\right)^{1-\gamma}\exp\left\{-0.5\left(1-\theta\right)\left(1-\gamma\right)\left[\gamma+\theta-\theta\gamma\right]\sigma_\varepsilon^2\right\}}{1-\beta\left(1+\alpha_1\right)^{1-\gamma}}\right]^{\frac{1}{1-\gamma}} - 1 \qquad \text{(A.17)}$$

A.3 Example 3 - ARIMA-BN Process (Reis, 2009):

From the Beveridge-Nelson decomposition,

$$\begin{aligned} x_t^{BN} &= \psi\left(1\right)\sum_{j=0}^{t}\varepsilon_j + \sum_{j=0}^{t}\varphi_j\varepsilon_{t-j} \\ &= \left[\psi\left(1\right)+\varphi_t\right]\varepsilon_0 + \left[\psi\left(1\right)+\varphi_{t-1}\right]\varepsilon_1 + \cdots + \left[\psi\left(1\right)+\varphi_1\right]\varepsilon_{t-1} + \left[\psi\left(1\right)+\varphi_0\right]\varepsilon_t \\ &= \sum_{j=0}^{t}\left[\psi\left(1\right)+\varphi_{t-j}\right]\varepsilon_j \qquad \text{(A.18)} \end{aligned}$$

Since ε_0 is revealed at the end of $t=0$, $\mathbb{E}_0\left[x_t^{BN}\right]=0$. Hence,

$$\begin{aligned} \sigma_{x_t^{BN}}^2 &\equiv \mathbb{E}\left[\left(x_t^{BN}-\mathbb{E}_0\left[x_t^{BN}\right]\right)^2\right] = \mathbb{E}\left[\left(x_t^{BN}\right)^2\right] = \mathbb{E}\left[\sum_{j=0}^{t}\left[\psi\left(1\right)+\varphi_{t-j}\right]^2\varepsilon_j^2\right] \\ &= \sum_{j=0}^{t}\left[\psi\left(1\right)^2+2\psi\left(1\right)\varphi_{t-j}+\varphi_{t-j}^2\right]\sigma_\varepsilon^2 \\ &= \left(t+1\right)\psi\left(1\right)^2\sigma_\varepsilon^2 + 2\psi\left(1\right)\sum_{j=0}^{t}\varphi_{t-j}\sigma_\varepsilon^2 + \sum_{j=0}^{t}\varphi_{t-j}^2\sigma_\varepsilon^2 \qquad \text{(A.19)} \end{aligned}$$

which can be rewritten into (17).

Hence, for $\gamma = 1$,

$$\begin{aligned}\lambda^T &= \exp\left\{\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\sigma_t^2\right\} - 1 = \exp\left\{\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\left(\rho_0+\rho_1 t\right)\right\} - 1 \\ &= \exp\left\{\frac{1}{2}\left(\rho_0+\frac{\beta}{1-\beta}\rho_1\right)\right\} - 1 \end{aligned} \tag{A.20}$$

$$\lambda^B = \exp\left\{\theta\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\left(\rho_0+\rho_1 t\right)\right\} - 1 = \exp\left\{\frac{\theta}{2}\left(\rho_0+\frac{\beta}{1-\beta}\rho_1\right)\right\} - 1 \tag{A.21}$$

$$\lambda^R = \exp\left\{\left(1-\theta\right)\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\left(\rho_0+\rho_1 t\right)\right\} - 1 = \exp\left\{\frac{1-\theta}{2}\left(\rho_0+\frac{\beta}{1-\beta}\rho_1\right)\right\} - 1 \tag{A.22}$$

For $\gamma > 1$,

$$\begin{aligned}\lambda^T &= \left[\frac{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t}{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t\exp\left\{-0.5\gamma\left(1-\gamma\right)\left(\rho_0+\rho_1 t\right)\right\}}\right]^{\frac{1}{1-\gamma}} - 1 \\ &= \left[\frac{1}{\exp\left\{-0.5\gamma\left(1-\gamma\right)\rho_0\right\}}\right]^{\frac{1}{1-\gamma}} \\ &\quad\times\left[\frac{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t}{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\exp\left\{-0.5\gamma\left(1-\gamma\right)\rho_1\right\}\right]^t}\right]^{\frac{1}{1-\gamma}} \\ &= \exp\left\{0.5\gamma\rho_0\right\}\left[\frac{1-\beta\left(1+\alpha_1\right)^{1-\gamma}\exp\left\{-0.5\gamma\left(1-\gamma\right)\rho_1\right\}}{1-\beta\left(1+\alpha_1\right)^{1-\gamma}}\right]^{\frac{1}{1-\gamma}}\end{aligned} \tag{A.23}$$

$$
\begin{aligned}
\lambda^{B} &= \left[\frac{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^{t}\exp\left\{-0.5\left(1-\gamma\right)\left(1-\theta\right)\left(\theta+\gamma-\gamma\theta\right)\left(\rho_0+\rho_1 t\right)\right\}}{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^{t}\exp\left\{-0.5\gamma\left(1-\gamma\right)\left(\rho_0+\rho_1 t\right)\right\}}\right]^{\frac{1}{1-\gamma}}-1 \\
&= \left[\frac{\exp\left\{-0.5\left(1-\gamma\right)\left(1-\theta\right)\left(\theta+\gamma-\gamma\theta\right)\rho_0\right\}}{\exp\left\{-0.5\gamma\left(1-\gamma\right)\rho_0\right\}}\right]^{\frac{1}{1-\gamma}} \\
&\quad\times\left[\frac{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\exp\left\{-0.5\left(1-\gamma\right)\left(1-\theta\right)\left(\theta+\gamma-\gamma\theta\right)\rho_1\right\}\right]^{t}}{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\exp\left\{-0.5\gamma\left(1-\gamma\right)\rho_1\right\}\right]^{t}}\right]^{\frac{1}{1-\gamma}}-1 \\
&= \frac{\exp\left\{0.5\gamma\rho_0\right\}}{\exp\left\{0.5\left(1-\theta\right)\left(\theta+\gamma-\gamma\theta\right)\rho_0\right\}} \\
&\quad\times\left[\frac{1-\beta\left(1+\alpha_1\right)^{1-\gamma}\exp\left\{-0.5\gamma\left(1-\gamma\right)\rho_1\right\}}{1-\beta\left(1+\alpha_1\right)^{1-\gamma}\exp\left\{-0.5\left(1-\gamma\right)\left(1-\theta\right)\left(\theta+\gamma-\gamma\theta\right)\rho_1\right\}}\right]^{\frac{1}{1-\gamma}}-1 \qquad \text{(A.24)}
\end{aligned}
$$

$$
\begin{aligned}
\lambda^{R} &= \left[\frac{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^{t}}{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^{t}\exp\left\{-0.5\left(1-\gamma\right)\left(1-\theta\right)\left(\theta+\gamma-\gamma\theta\right)\left(\rho_0+\rho_1 t\right)\right\}}\right]^{\frac{1}{1-\gamma}}-1 \\
&= \left[\frac{1}{\exp\left\{-0.5\left(1-\gamma\right)\left(1-\theta\right)\left(\theta+\gamma-\gamma\theta\right)\rho_0\right\}}\right]^{\frac{1}{1-\gamma}} \\
&\quad\times\left[\frac{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^{t}}{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\exp\left\{-0.5\left(1-\gamma\right)\left(1-\theta\right)\left(\theta+\gamma-\gamma\theta\right)\rho_1\right\}\right]^{t}}\right]^{\frac{1}{1-\gamma}}-1 \\
&= \exp\left\{0.5\left(1-\theta\right)\left(\theta+\gamma-\gamma\theta\right)\rho_0\right\} \\
&\quad\times\left[\frac{1-\beta\left(1+\alpha_1\right)^{1-\gamma}\exp\left\{-0.5\left(1-\gamma\right)\left(1-\theta\right)\left(\theta+\gamma-\gamma\theta\right)\rho_1\right\}}{1-\beta\left(1+\alpha_1\right)^{1-\gamma}}\right]^{\frac{1}{1-\gamma}}-1
\end{aligned}
$$

A.4 The Literature-based Cost λ^{lit}

Here we characterize in our three applications the welfare cost of business cycles in the absence of observed consumption as proposed in our decomposition. We simply substitute σ_ε^2 by σ_u^2 in our previous calculations and use the formula for λ^T for each type of shock. Recall that, in our methodology, $\sigma_u^2 = (1-\theta_2)^2\sigma_\varepsilon^2$.

Example 1 (Lucas, 1987) :

$$\lambda^{lit} = \begin{cases} \exp\left(\dfrac{\sigma_u^2}{2}\right) - 1, \text{ if } \gamma = 1 \\ \exp\left(\dfrac{\gamma\sigma_u^2}{2}\right) - 1, \text{ if } \gamma > 1 \end{cases} \tag{A.25}$$

Example 2 (Obstfeld, 1994) :

$$\lambda^{lit} = \begin{cases} \exp\left(\dfrac{\sigma_u^2}{2(1-\beta)}\right) - 1, \text{ if } \gamma = 1 \\ \exp\left\{0.5\gamma\sigma_u^2\right\}\left[\dfrac{1-\Gamma\exp -0.5\gamma(1-\gamma)\sigma_u^2}{1-\Gamma}\right]^{\frac{1}{1-\gamma}} - 1, \text{ if } \gamma > 1 \end{cases} \tag{A.26}$$

Example 3 - ARIMA-BN Process (Reis, 2009):

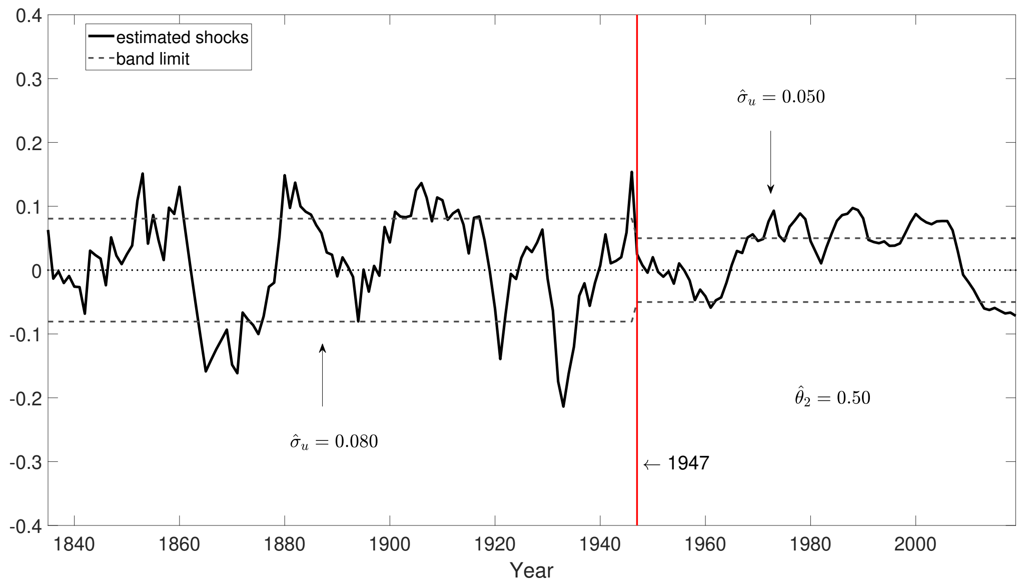
In this case, the substitution is in equation (17).

$$\lambda^{lit} = \begin{cases} \exp\left\{\dfrac{1}{2}\left(\rho_0 + \dfrac{\beta}{1-\beta}\rho_1\right)\right\} - 1, & \text{if } \gamma = 1 \\ \exp\left\{0.5\gamma\rho_0\right\}\left[\dfrac{1-\Gamma\exp\left\{-0.5\gamma\left(1-\gamma\right)\rho_1\right\}}{1-\Gamma}\right]^{\dfrac{1}{1-\gamma}}, & \text{if } \gamma > 1 \end{cases} \tag{A.27}$$

B Extra Material

B.1 Extra Material on the Identification

Figure A.1: Estimated residuals of transitory shocks between 1835 and 2019.



Notes: The figure shows the time series for per capita consumption for the US between 1835 and 2019 with our augmented sample of the Barro and Ursúa (2010) data. The vertical line marks the year 1947, at the end of WWII. We report the standard errors for the two sub-periods generated by this line along with the average and band limits equivalent to $2\sigma_u$.

B.2 Extra Material on the Main Estimation

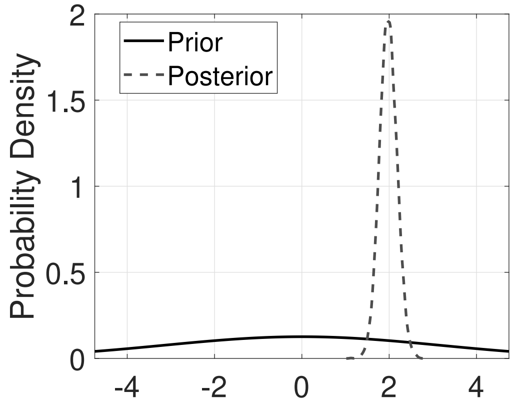
Table A.1: Controling for consumption shares - Estimation.

	Transitory Shocks		Permanent Shocks		ARIMA-BN
	1930-1946	1947 - 2019	1930-1946	1947 - 2019	1947 - 2019
Nondurables share	-2.405	-3.739	-0.0355	-1.034	-1.066
	(1.351)	(0.374)	(0.545)	(0.161)	(0.172)
Services share	-2.974	-2.560	-1.764	-0.916	-0.946
	(0.824)	(0.490)	(0.487)	(0.141)	(0.152)
$\hat{\pi}_0$	11.06	11.44	0.813	0.850	0.878
	(0.837)	(0.337)	(0.432)	(0.128)	(0.138)
$\hat{\pi}_1$	0.0213	0.0190	-	-	-
	(0.00497)	(0.00118)	-	-	-
$\hat{\phi}_1$	-	-	-	-	-0.0501
	-	-	-	-	(0.0883)
$\hat{\sigma}_u^2$	0.0029	0.0008	0.0011	0.00018	0.00018

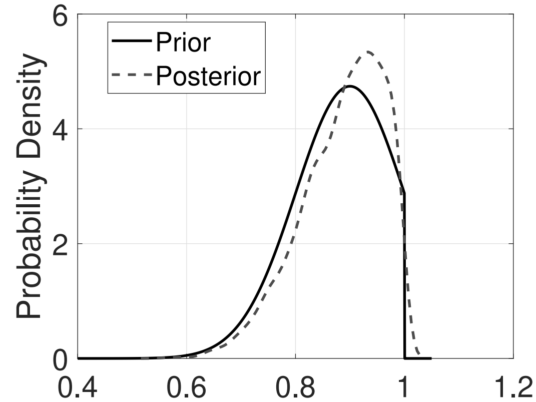
Notes: The table displays the estimated parameters obtained by running regression (22) for transitory shocks and regression (25) for permanent shocks and the ARIMA-BN process with added controls for the share of nondurable goods and services. The time series used is the sample obtained from the BEA's NIPA Table 2.3.5. US Bureau of Economic Analysis (2021).

B.3 Extra Material on the Estimation of the Time-Varying θ

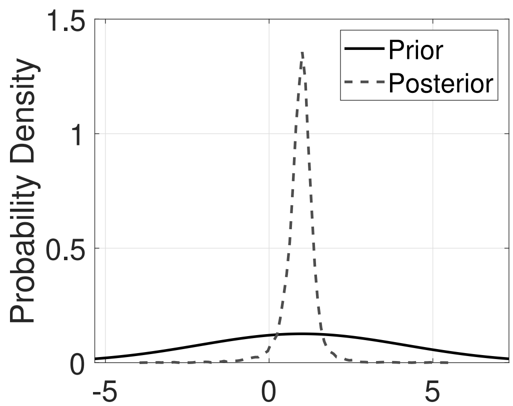
Figure A.2: Prior and Posterior Distributions



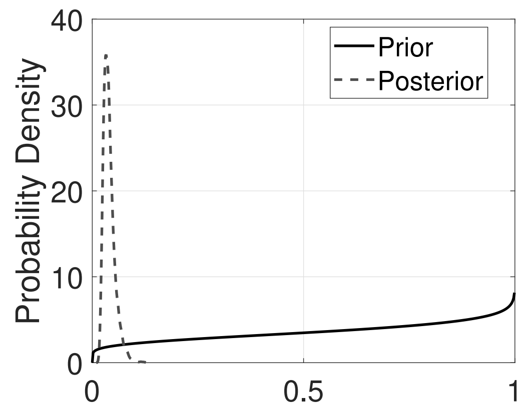
(a) π_0



(b) ϕ_h



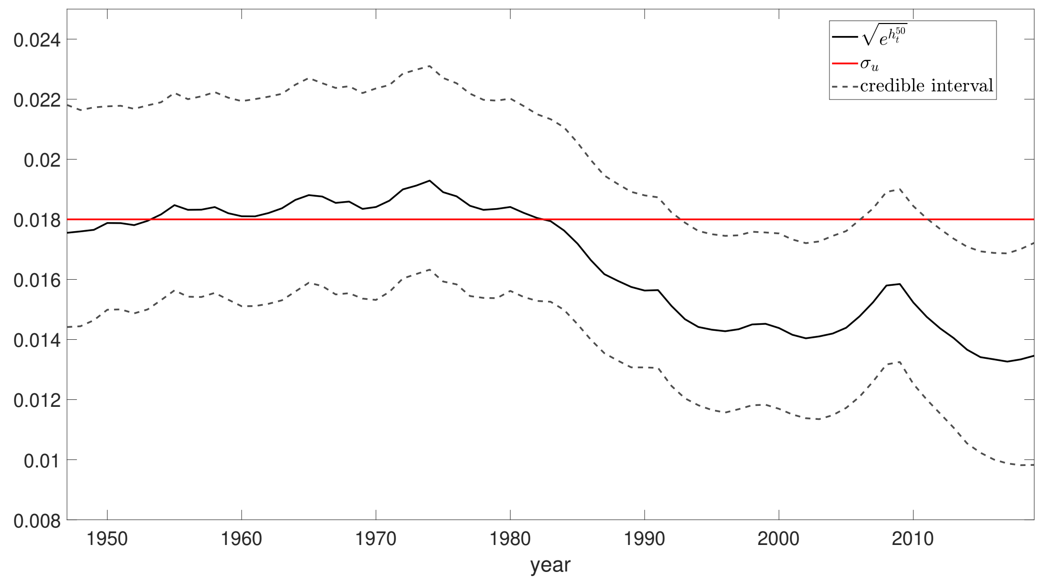
(c) μ_h



(d) ω_h^2

Notes: The figure shows the prior and posterior distributions for the four parameters estimated for the stochastic volatility model. In each graph, the solid black line represents the density function of the prior distribution and the dashed line represents the density function of the posterior distribution. The estimation used the MCMC approach developed in Chan and Grant (2016). The details of the model can be found in Section 6.2.

Figure A.3: Estimated $\sqrt{e^{h_t^{50}}}$ for the stochastic volatility model.



Notes: The figure shows the estimated time series for $\sqrt{e^{h_t^{50}}}$. The solid black line shows the values associated with the median quantile of the estimation, while the dashed lines indicating the bands of the credible interval are associated with quantiles 16 and 84. The solid red line shows the $\hat{\sigma}_u$ obtained in the estimation shown in Table 1. These values were obtained from the estimation of the stochastic volatility process for the first difference of log-consumption described in equations (31) and (32). The series spans from 1947 through 2019 and is computed considering $\theta_1 = 0.2$ and $\hat{\sigma}_{u,1} = 0.0021$.

C Estimates of Welfare Costs for Different β's

C.1 Full Sample

In Tables A.2 and A.3 we present our estimations of the welfare cost using the full sample as in the main text. For the case of permanent and ARIMA-BN shocks, we compute the λ's for different values of β.

Table A.2: Welfare cost - Full sample

Transitory shocks													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.37	0.44	0.50	0.56	0.37	0.44	0.50	0.56	0.37	0.44	0.50	0.56	-
$\gamma = 1$	0.32	0.40	0.51	0.66	0.12	0.17	0.25	0.37	0.20	0.23	0.25	0.29	0.13
$\gamma = 2.5$	0.81	1.00	1.27	1.66	0.42	0.58	0.82	1.17	0.39	0.42	0.44	0.48	0.32
$\gamma = 5$	1.63	2.01	2.55	3.35	0.91	1.27	1.78	2.53	0.71	0.74	0.76	0.80	0.64
$\gamma = 7.5$	2.45	3.04	3.86	5.07	1.40	1.96	2.74	3.90	1.03	1.06	1.08	1.12	0.96
Permanent shocks													
$\beta = 0.95$													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.63	0.67	0.70	0.74	0.63	0.67	0.70	0.74	0.63	0.67	0.70	0.74	-
$\gamma = 1$	2.10	2.60	3.30	4.33	1.31	1.72	2.31	3.18	0.77	0.86	0.97	1.11	0.29
$\gamma = 2.5$	3.42	4.26	5.46	7.27	2.63	3.41	4.53	6.23	0.77	0.82	0.89	0.98	0.46
$\gamma = 5$	4.58	5.79	7.60	10.51	3.77	4.94	6.69	9.51	0.78	0.81	0.86	0.91	0.58
$\gamma = 7.5$	5.44	7.03	9.56	14.12	4.61	6.16	8.63	13.11	0.80	0.82	0.86	0.90	0.65
$\beta = 0.96$													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.63	0.67	0.70	0.74	0.63	0.67	0.70	0.74	0.63	0.67	0.70	0.74	-
$\gamma = 1$	2.63	3.92	4.99	6.65	1.64	3.02	4.11	5.74	0.97	0.88	0.84	0.86	0.36
$\gamma = 2.5$	3.25	4.89	6.33	8.91	2.15	3.92	5.40	7.97	1.08	0.94	0.88	0.88	0.52
$\gamma = 5$	4.14	6.28	8.34	13.01	2.89	5.21	7.34	11.99	1.21	1.02	0.93	0.90	0.63
$\gamma = 7.5$	5.44	8.39	11.60	15.47	4.00	7.19	10.52	14.39	1.39	1.12	0.98	0.95	0.69
$\beta = 0.97$													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.63	0.67	0.70	0.74	0.63	0.67	0.70	0.74	0.63	0.67	0.70	0.74	-
$\gamma = 1$	3.52	4.36	5.55	7.31	2.20	2.88	3.87	5.36	1.29	1.44	1.62	1.85	0.48
$\gamma = 2.5$	4.60	5.74	7.40	9.92	3.54	4.60	6.14	8.51	1.02	1.09	1.18	1.30	0.61
$\gamma = 5$	5.48	6.97	9.23	12.96	4.52	5.96	8.14	11.76	0.92	0.96	1.01	1.07	0.69
$\gamma = 7.5$	6.23	8.13	11.21	17.13	5.29	7.14	10.16	15.97	0.89	0.92	0.96	1.00	0.73

Notes: The table displays the computed parameters for the decomposition of the welfare cost of total economic fluctuations expanding the one in the main text for different β's. The numbers are obtained using equations (10) through (15) with the estimates shown in Table 1. All of the entries are in percentages of lifetime consumption. We also report an extra welfare cost measure, λ^{lit}, described in Appendix A.4. We report numbers for the relative degree of risk aversion $\gamma \in \{1, 2.5, 5, 7.5\}$, for the implied $\hat{\theta}_2$ along the grid for $\theta_1 \in \{0, 0.1, 0.2, 0.3\}$, and with $\beta \in \{0.95, 0.96, 0.97\}$ for the permanent shocks.

Table A.3: Welfare cost - Full sample - ARIMA-BN

$\beta = 0.95$													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.61	0.65	0.69	0.73	0.61	0.65	0.69	0.73	0.61	0.65	0.69	0.73	-
$\gamma = 1$	4.07	5.05	6.43	8.48	2.48	3.27	4.40	6.12	1.55	1.72	1.94	2.22	0.60
$\gamma = 2.5$	5.91	7.42	9.63	13.05	4.49	5.88	7.92	11.13	1.36	1.46	1.58	1.73	0.83
$\gamma = 5$	7.50	9.72	13.29	19.77	6.16	8.30	11.74	18.03	1.26	1.32	1.38	1.47	0.96
$\gamma = 7.5$	8.99	12.35	18.89	40.20	7.68	10.95	17.35	38.31	1.22	1.26	1.31	1.37	1.01
$\beta = 0.96$													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.61	0.65	0.69	0.73	0.61	0.65	0.69	0.73	0.61	0.65	0.69	0.73	-
$\gamma = 1$	5.15	6.40	8.16	10.79	3.13	4.13	5.58	7.77	1.96	2.18	2.45	2.81	0.75
$\gamma = 2.5$	6.75	8.49	11.06	15.08	5.13	6.73	9.11	12.86	1.54	1.65	1.79	1.96	0.94
$\gamma = 5$	8.16	10.63	14.65	22.16	6.71	9.09	12.96	20.26	1.36	1.41	1.49	1.58	1.03
$\gamma = 7.5$	9.64	13.37	20.92	51.68	8.25	11.88	19.28	49.52	1.29	1.33	1.38	1.44	1.06
$\beta = 0.97$													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.61	0.65	0.69	0.73	0.61	0.65	0.69	0.73	0.61	0.65	0.69	0.73	-
$\gamma = 1$	6.98	8.69	11.12	14.76	4.23	5.59	7.56	10.57	2.64	2.93	3.31	3.79	1.01
$\gamma = 2.5$	7.84	9.91	12.95	17.80	5.96	7.86	10.68	15.19	1.78	1.90	2.06	2.26	1.08
$\gamma = 5$	8.93	11.71	16.29	25.19	7.36	10.03	14.46	23.09	1.46	1.53	1.61	1.71	1.11
$\gamma = 7.5$	10.38	14.56	23.47	85.20	8.90	12.97	21.70	82.42	1.36	1.40	1.45	1.52	1.12

Notes: The table displays the computed parameters for the decomposition of the welfare cost of total economic fluctuations expanding the one in the main text for different β's. The numbers are obtained using equations (18) through (20) with the estimates shown in Table 1. All of the entries are in percentages of lifetime consumption. We also report an extra welfare cost measure, λ^{lit}, described in Appendix A.4. We report numbers for the relative degree of risk aversion $\gamma \in \{1, 2.5, 5, 7.5\}$, for the implied $\hat{\theta}_2$ along the grid for $\theta_1 \in \{0, 0.1, 0.2, 0.3\}$, and with $\beta \in \{0.95, 0.96, 0.97\}$ for the ARIMA-BN process.

C.2 Structural Break

In Tables A.4 and A.5 we present our estimations of the welfare cost using the sample adjusted for structural breaks as described in the main text. For the case of permanent and ARIMA-BN shocks, we compute the λ's for different values of β.

Table A.4: Structural break - Welfare cost

Transitory shocks													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.32	0.39	0.46	0.53	0.32	0.39	0.46	0.53	0.32	0.39	0.46	0.53	-
$\gamma = 1$	0.28	0.34	0.43	0.56	0.09	0.13	0.20	0.30	0.19	0.21	0.23	0.27	0.13
$\gamma = 2.5$	0.69	0.85	1.08	1.42	0.31	0.45	0.65	0.95	0.38	0.40	0.43	0.46	0.32
$\gamma = 5$	1.39	1.72	2.18	2.85	0.69	0.99	1.42	2.06	0.70	0.72	0.74	0.78	0.64
$\gamma = 7.5$	2.09	2.58	3.28	4.31	1.06	1.53	2.20	3.18	1.02	1.04	1.06	1.10	0.96
Permanent shocks													
$\beta = 0.95$													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.62	0.65	0.69	0.73	0.62	0.65	0.69	0.73	0.62	0.65	0.69	0.73	-
$\gamma = 1$	1.96	2.43	3.08	4.04	1.20	1.58	2.12	2.94	0.75	0.83	0.94	1.07	0.29
$\gamma = 2.5$	3.19	3.97	5.09	6.76	2.42	3.14	4.18	5.75	0.75	0.80	0.87	0.96	0.46
$\gamma = 5$	4.25	5.37	7.03	9.67	3.46	4.53	6.13	8.69	0.77	0.80	0.84	0.90	0.58
$\gamma = 7.5$	4.91	6.32	8.51	12.36	4.09	5.46	7.61	11.38	0.79	0.81	0.84	0.88	0.65
$\beta = 0.96$													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.62	0.65	0.69	0.73	0.62	0.65	0.69	0.73	0.62	0.65	0.69	0.73	-
$\gamma = 1$	2.46	3.66	4.63	6.09	1.51	2.78	3.77	5.20	0.94	0.86	0.83	0.85	0.36
$\gamma = 2.5$	3.04	4.56	5.86	8.09	1.98	3.61	4.95	7.16	1.04	0.92	0.87	0.87	0.52
$\gamma = 5$	3.86	5.85	7.70	11.59	2.66	4.81	6.73	10.60	1.17	0.99	0.91	0.90	0.63
$\gamma = 7.5$	4.98	7.63	10.40	13.46	3.60	6.48	9.34	12.41	1.33	1.08	0.96	0.93	0.69
$\beta = 0.97$													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.62	0.65	0.69	0.73	0.62	0.65	0.69	0.73	0.62	0.65	0.69	0.73	-
$\gamma = 1$	3.29	4.08	5.19	6.83	2.01	2.65	3.56	4.95	1.25	1.39	1.57	1.79	0.48
$\gamma = 2.5$	4.29	5.35	6.88	9.21	3.25	4.24	5.66	7.84	1.00	1.07	1.16	1.27	0.61
$\gamma = 5$	5.08	6.45	8.51	11.87	4.14	5.46	7.45	10.71	0.90	0.94	0.99	1.05	0.69
$\gamma = 7.5$	5.61	7.27	9.92	14.79	4.69	6.30	8.90	13.67	0.88	0.91	0.94	0.99	0.73

Notes: The table displays the computed parameters for the decomposition of the welfare cost of total economic fluctuations using a robustness sample that avoids the structural break in 1931. The numbers are obtained using equations (10) through (15) with the estimates shown in Table 4. All of the entries are in percentages of lifetime consumption. We also report an extra welfare cost measure, λ^{lit}, described in Appendix A.4. We report numbers for the relative degree of risk aversion $\gamma \in \{1, 2.5, 5, 7.5\}$, for the implied $\hat{\theta}_2$ along the grid for $\theta_1 \in \{0, 0.1, 0.2, 0.3\}$, and with $\beta \in \{0.95, 0.96, 0.97\}$ for the permanent shocks.

Table A.5: Structural break - Welfare cost - ARIMA-BN

	$\beta = 0.95$												
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.60	0.64	0.68	0.72	0.60	0.64	0.68	0.72	0.60	0.64	0.68	0.72	-
$\gamma = 1$	3.80	4.71	6.00	7.91	2.27	3.00	4.05	5.64	1.50	1.67	1.88	2.15	0.60
$\gamma = 2.5$	5.50	6.90	8.94	12.08	4.12	5.40	7.28	10.21	1.33	1.43	1.54	1.69	0.83
$\gamma = 5$	6.92	8.94	12.13	17.79	5.60	7.54	10.62	16.11	1.25	1.30	1.36	1.45	0.96
$\gamma = 7.5$	8.19	11.12	16.56	31.28	6.90	9.75	15.07	29.52	1.21	1.25	1.30	1.35	1.01
	$\beta = 0.96$												
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.60	0.64	0.68	0.72	0.60	0.64	0.68	0.72	0.60	0.64	0.68	0.72	-
$\gamma = 1$	4.81	5.97	7.62	10.06	2.86	3.79	5.13	7.16	1.89	2.10	2.37	2.71	0.75
$\gamma = 2.5$	6.28	7.89	10.25	13.93	4.70	6.18	8.36	11.79	1.51	1.62	1.75	1.92	0.94
$\gamma = 5$	7.52	9.76	13.33	19.83	6.10	8.25	11.69	18.00	1.34	1.40	1.47	1.56	1.03
$\gamma = 7.5$	8.76	11.99	18.17	37.05	7.39	10.53	16.58	35.12	1.28	1.31	1.36	1.43	1.06
	$\beta = 0.97$												
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.60	0.64	0.68	0.72	0.60	0.64	0.68	0.72	0.60	0.64	0.68	0.72	-
$\gamma = 1$	6.51	8.10	10.36	13.75	3.87	5.12	6.95	9.73	2.55	2.84	3.20	3.66	1.01
$\gamma = 2.5$	7.29	9.19	11.99	16.41	5.46	7.20	9.78	13.90	1.74	1.86	2.01	2.21	1.08
$\gamma = 5$	8.22	10.73	14.78	22.39	6.68	9.08	12.99	20.37	1.44	1.51	1.58	1.68	1.11
$\gamma = 7.5$	9.41	13.00	20.15	46.47	7.96	11.45	18.44	44.30	1.35	1.39	1.44	1.50	1.12

Notes: The table displays the computed parameters for the decomposition of the welfare cost of total economic fluctuations using a robustness sample that avoids the structural break in 1931. The numbers are obtained using equations (18) through (20) with the estimates shown in Table 4. All of the entries are in percentages of lifetime consumption. We also report an extra welfare cost measure, λ^{lit}, described in Appendix A.4. We report numbers for the relative degree of risk aversion $\gamma \in \{1, 2.5, 5, 7.5\}$, for the implied $\hat{\theta}_2$ along the grid for $\theta_1 \in \{0, 0.1, 0.2, 0.3\}$, and with $\beta \in \{0.95, 0.96, 0.97\}$ for the ARIMA-BN process.

D Extra Robustness Exercises

D.1 Removing the Interwar Period

As the previous exercise used the disjoint periods (1835-1930 and 1947-2019), we run an additional experiment where we use the 1931 break in the time series as a reference point to design two new intervals. In the previous exercise, we have removed 15 periods - years 1931 to 1946 - from the full sample. Those periods were exclusively defined after the break. For this case, we remove a similar interval for the period before the 1931 break. We construct two sub-samples by excluding the interwar period from our data, which results in a first period with years 1835 to 1913 and a second period from 1947 to 2019, the last one as in our main analysis.[26]

Besides the structural break in the consumption series during the interwar period, many other relevant macroeconomic events happened during this window of time. For example, we have the 1929 crisis and the Great Depression that followed. In general, this period was marked by highly unstable macroeconomic outcomes, and hence, it is worth subtracting it from the sample to better measure pre-war volatility. Once again, the results are similar to those of our original analysis. Table A.6 presents the estimated and implied parameters and Tables A.7 and A.8 present the computed λ's using the estimations in Table A.6. Similarly to the robustness check with structural breaks, there is no substantial change in the results, with our preferred total cost being roughly 1 percentage point smaller than the one shown in our main exercise.

[26]We also run an experiment by removing exactly 15 periods before and after the break, that is, using sub-samples from 1835-1915 and 1947-2019. As expected, the results are so similar to the results in this subsection that we only report the exercise where we remove the interwar period.

Table A.6: Removing the interwar period - Estimation

Transitory shocks				Permanent shocks				ARIMA-BN			
Estimated parameters				Estimated parameters				Estimated parameters			
	$\hat{\pi}_0$	$\hat{\pi}_1$	$\hat{\sigma}^2_u$		$\hat{\pi}_0$	$\hat{\pi}_1$	$\hat{\sigma}^2_u$		$\hat{\pi}_0$	$\hat{\phi}_1$	$\hat{\sigma}^2_u$
1835 - 1913	7.7099	0.0119	0.0057	1835 - 1913	0.0107	-	0.0019	1835 - 1913	0.0107	-	0.0019
	(.0175)	(.0004)			(.0049)				(.0049)		
1947 - 2019	6.6156	0.0219	0.0025	1947 - 2019	0.0203	-	0.0003	1947 - 2019	0.0189	0.2919	0.0003
	(.0427)	(.0003)			(.0020)				(.0030)	(.1321)	
Implied parameters				Implied parameters				Implied parameters			
θ_1	$\hat{\theta}_2$	$\hat{\alpha}_1$	$\hat{\sigma}^2_\varepsilon$	θ_1	$\hat{\theta}_2$	$\hat{\alpha}_1$	$\hat{\sigma}^2_\varepsilon$	θ_1	$\hat{\theta}_2$	$\hat{\alpha}_1$	$\hat{\sigma}^2_\varepsilon$
0.00	0.3309	0.0222	0.0057	0.00	0.6120	0.0208	0.0019	0.00	0.5972	0.0277	0.0019
0.10	0.3978	0.0222	0.0070	0.10	0.6508	0.0209	0.0023	0.10	0.6375	0.0277	0.0023
0.20	0.4647	0.0222	0.0089	0.20	0.6896	0.0209	0.0030	0.20	0.6777	0.0278	0.0030
0.30	0.5316	0.0222	0.0116	0.30	0.7284	0.0210	0.0039	0.30	0.7180	0.0279	0.0039

Notes: The table displays the estimated parameters obtained by running regression (22) for transitory shocks and regression (25) for permanent and ARIMA-BN shocks. The time series used is our sample of the augmented Barro and Ursúa (2010) data excluding the interwar period. The series is I(1) as identified by the ADF, PP, and DF-GLS tests. The first-difference of the series is identified, by both the AIC and the BIC criteria, as an ARMA (0,0) for the pre-1947 period and an ARMA(1,0) for the subsequent years. The implied parameters are obtained using the formulas described in the text and equation (30).

Table A.7: Removing the interwar period - Welfare cost

	λ^T				λ^B				λ^R				λ^{lit}
Transitory shocks													
$\hat{\theta}_2$	0.33	0.40	0.46	0.53	0.33	0.40	0.46	0.53	0.33	0.40	0.46	0.53	-
$\gamma = 1$	0.28	0.35	0.44	0.58	0.09	0.14	0.21	0.31	0.19	0.21	0.24	0.27	0.13
$\gamma = 2.5$	0.71	0.88	1.11	1.46	0.33	0.47	0.68	0.99	0.38	0.40	0.43	0.46	0.32
$\gamma = 5$	1.43	1.76	2.24	2.93	0.72	1.04	1.48	2.14	0.70	0.72	0.75	0.78	0.64
$\gamma = 7.5$	2.15	2.66	3.38	4.43	1.12	1.60	2.29	3.30	1.02	1.04	1.07	1.10	0.96
Permanent shocks													
$\beta = 0.95$													
$\hat{\theta}_2$	0.61	0.65	0.69	0.73	0.61	0.65	0.69	0.73	0.61	0.65	0.69	0.73	-
$\gamma = 1$	1.92	2.38	3.02	3.96	1.17	1.54	2.07	2.87	0.74	0.82	0.93	1.06	0.29
$\gamma = 2.5$	3.13	3.89	4.98	6.62	2.36	3.07	4.08	5.62	0.75	0.80	0.86	0.95	0.46
$\gamma = 5$	4.16	5.25	6.87	9.44	3.37	4.42	5.98	8.47	0.77	0.80	0.84	0.89	0.58
$\gamma = 7.5$	4.91	6.32	8.51	12.36	4.09	5.46	7.61	11.38	0.79	0.81	0.84	0.88	0.65
$\beta = 0.96$													
$\hat{\theta}_2$	0.61	0.65	0.69	0.73	0.61	0.65	0.69	0.73	0.61	0.65	0.69	0.73	-
$\gamma = 1$	2.41	3.58	4.53	5.94	1.47	2.71	3.67	5.05	0.93	0.85	0.83	0.85	0.36
$\gamma = 2.5$	2.98	4.47	5.73	7.87	1.93	3.52	4.83	6.94	1.03	0.91	0.86	0.87	0.52
$\gamma = 5$	3.79	5.73	7.52	11.22	2.60	4.69	6.55	10.24	1.16	0.99	0.91	0.89	0.63
$\gamma = 7.5$	4.98	7.63	10.40	13.46	3.60	6.48	9.34	12.41	1.33	1.08	0.96	0.93	0.69
$\beta = 0.97$													
$\hat{\theta}_2$	0.61	0.65	0.69	0.73	0.61	0.65	0.69	0.73	0.61	0.65	0.69	0.73	-
$\gamma = 1$	3.22	3.99	5.08	6.69	1.96	2.58	3.48	4.83	1.24	1.38	1.55	1.77	0.48
$\gamma = 2.5$	4.20	5.24	6.74	9.01	3.17	4.13	5.52	7.65	0.99	1.06	1.15	1.26	0.61
$\gamma = 5$	4.97	6.31	8.31	11.58	4.04	5.32	7.25	10.42	0.90	0.94	0.99	1.05	0.69
$\gamma = 7.5$	5.61	7.27	9.92	14.79	4.69	6.30	8.90	13.67	0.88	0.91	0.94	0.99	0.73

Notes: The table displays the computed parameters for the decomposition of the welfare cost of total economic fluctuations using a robustness sample that excludes the interwar period. The numbers are obtained using equations (10) through (15) with the estimates shown in Table A.6. All measures are in percentages of lifetime consumption. We also report an extra welfare cost measure, λ^{lit}, described in Appendix A.4. We report numbers for the relative degree of risk aversion $\gamma \in \{1, 2.5, 5, 7.5\}$, for the implied $\hat{\theta}_2$ along the grid for $\theta_1 \in \{0, 0.1, 0.2, 0.3\}$, and with $\beta \in \{0.95, 0.96, 0.97\}$ for the permanent shocks.

Table A.8: Removing the interwar period - Welfare cost - ARIMA-BN

	λ^T				λ^B				λ^R				λ^{lit}
$\beta = 0.95$													
$\hat{\theta}_2$	0.60	0.64	0.68	0.72	0.60	0.64	0.68	0.72	0.60	0.64	0.68	0.72	-
$\gamma = 1$	3.73	4.62	5.88	7.75	2.21	2.92	3.95	5.51	1.48	1.65	1.86	2.13	0.60
$\gamma = 2.5$	5.39	6.76	8.74	11.81	4.01	5.26	7.10	9.96	1.32	1.42	1.53	1.68	0.83
$\gamma = 5$	6.76	8.72	11.81	17.26	5.45	7.34	10.31	15.59	1.24	1.29	1.36	1.44	0.96
$\gamma = 7.5$	7.97	10.79	15.96	29.41	6.68	9.42	14.48	27.68	1.21	1.25	1.29	1.35	1.01
$\beta = 0.96$													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.60	0.64	0.68	0.72	0.60	0.64	0.68	0.72	0.60	0.64	0.68	0.72	-
$\gamma = 1$	4.71	5.85	7.46	9.86	2.79	3.69	5.00	6.98	1.87	2.08	2.35	2.69	0.75
$\gamma = 2.5$	6.15	7.72	10.03	13.61	4.58	6.02	8.15	11.49	1.50	1.61	1.74	1.90	0.94
$\gamma = 5$	7.34	9.52	12.97	19.22	5.93	8.02	11.35	17.40	1.33	1.39	1.46	1.55	1.03
$\gamma = 7.5$	8.52	11.62	17.48	34.39	7.16	10.18	15.90	32.50	1.27	1.31	1.36	1.42	1.06
$\beta = 0.97$													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.60	0.64	0.68	0.72	0.60	0.64	0.68	0.72	0.60	0.64	0.68	0.72	-
$\gamma = 1$	6.38	7.94	10.15	13.46	3.76	4.99	6.77	9.49	2.52	2.81	3.16	3.63	1.01
$\gamma = 2.5$	7.14	8.99	11.73	16.03	5.32	7.02	9.54	13.54	1.73	1.85	2.00	2.19	1.08
$\gamma = 5$	8.03	10.46	14.37	21.65	6.50	8.82	12.60	19.65	1.44	1.50	1.58	1.67	1.11
$\gamma = 7.5$	9.15	12.58	19.32	42.06	7.70	11.05	17.64	39.96	1.34	1.38	1.43	1.50	1.12

Notes: The table displays the computed parameters for the decomposition of the welfare cost of total economic fluctuations using a robustness sample that excludes the interwar period. The numbers are obtained using equations (18) through (20) with the estimates shown in Table A.6. All measures are in percentages of lifetime consumption. We also report an extra welfare cost measure, λ^{lit}, described in Appendix A.4. We report numbers for the relative degree of risk aversion $\gamma \in \{1, 2.5, 5, 7.5\}$, for the implied $\hat{\theta}_2$ along the grid for $\theta_1 \in \{0, 0.1, 0.2, 0.3\}$, and with $\beta \in \{0.95, 0.96, 0.97\}$ for the ARIMA-BN process.

D.2 Using Barro and Ursúa's (2010) Sample Data

We now show our results using Barro and Ursúa's (2010) data only. Table A.9 shows the regression estimates and Tables A.10, A.11, and A.12 show the calculation of the welfare costs for all types of shocks and for different β's.

There are some differences with the results of the analysis in the main text that require qualification. The first occurs in the case of transitory shocks, where we observe a lower volatility in the post-war period when compared to the augmented sample. This leads to a significant increase in the span of stabilization power and thus a higher welfare benefit of ongoing policies. Since this is not our preferred structure for the shocks, we understand that it does not make a substantial difference for our main findings.

The second difference is that for the case of ARIMA-BN shocks, the sample is also well-modeled by an ARIMA(0,1,1), yielding two different sets of welfare costs for these shocks, shown in Tables A.11 and A.12. If we compare the results for the ARIMA(1,1,0) depicted in Table A.11, which is the one equivalent to the analysis in our main text, we observe that the total cost of economic fluctuations is roughly 1 percentage point smaller, a difference that is not substantial for the main message of our analysis.

Table A.9: Barro and Ursúa (2010) Data - Estimation

Transitory shocks				Permanent shocks				ARIMA			
Estimated parameters				Estimated parameters				Estimated parameters			
	$\hat{\pi}_0$	$\hat{\pi}_1$	$\hat{\sigma}_u^2$		$\hat{\pi}_0$	$\hat{\pi}_1$	$\hat{\sigma}_u^2$		$\hat{\pi}_0$	$\hat{\phi}_1$	$\hat{\sigma}_u^2$
1835 - 1946	7.7417	0.0109	0.0065	1835 - 1946	0.0113	-	0.0021	1835 - 1946	0.0113	-	0.0021
	(.0156)	(.0002)			(.0043)				(.0043)		
1947 - 2009	6.4230	0.0233	0.0010	1947 - 2009	0.0210	-	0.0003	1947 - 2009	0.0207	0.2674	0.0003
	(.0330)	(.0002)			(.0023)				(.0030)	(.1316)	
Implied parameters				Implied parameters				Implied parameters			
θ_1	$\hat{\theta}_2$	$\hat{\alpha}_1$	$\hat{\sigma}_\varepsilon^2$	θ_1	$\hat{\theta}_2$	$\hat{\alpha}_1$	$\hat{\sigma}_\varepsilon^2$	θ_1	$\hat{\theta}_2$	$\hat{\alpha}_1$	$\hat{\sigma}_\varepsilon^2$
0.00	0.5976	0.0236	0.0065	0.00	0.6058	0.0217	0.0021	0.00	0.6223	0.0292	0.0021
0.10	0.6378	0.0236	0.0080	0.10	0.6453	0.0217	0.0026	0.10	0.6600	0.0293	0.0026
0.20	0.6781	0.0236	0.0101	0.20	0.6847	0.0218	0.0032	0.20	0.6978	0.0293	0.0032
0.30	0.7183	0.0236	0.0132	0.30	0.7241	0.0218	0.0042	0.30	0.7356	0.0294	0.0042

Notes: The table displays the estimated parameters obtained by running regression (22) for transitory shocks and regression (25) for permanent shocks. The time series used is the Barro and Ursúa (2010) data. The implied parameters are obtained using the formulas described in the text and equation (30).

Table A.10: Barro and Ursúa (2008) Data - Welfare Cost

Transitory shocks													
	λ^T				λ^B				λ^R				λ
$\hat{\theta}_2$	0.60	0.64	0.68	0.72	0.60	0.64	0.68	0.72	0.60	0.64	0.68	0.72	-
$\gamma = 1$	0.32	0.40	0.51	0.66	0.19	0.25	0.34	0.47	0.13	0.14	0.16	0.19	0.05
$\gamma = 2.5$	0.81	1.00	1.27	1.66	0.60	0.78	1.03	1.39	0.21	0.22	0.24	0.26	0.13
$\gamma = 5$	1.63	2.01	2.55	3.35	1.28	1.65	2.17	2.94	0.34	0.35	0.37	0.40	0.26
$\gamma = 7.5$	2.45	3.04	3.86	5.07	1.97	2.54	3.34	4.52	0.47	0.49	0.50	0.53	0.39
Permanent shocks													
$\beta = 0.95$													
	λ^T				λ^B				λ^R				λ
$\hat{\theta}_2$	0.54	0.58	0.63	0.68	0.54	0.58	0.63	0.68	0.54	0.58	0.63	0.68	-
$\gamma = 1$	2.10	2.60	3.30	4.33	1.27	1.67	2.25	3.11	0.82	0.91	1.03	1.18	0.32
$\gamma = 2.5$	3.37	4.20	5.38	7.16	2.53	3.29	4.39	6.05	0.82	0.88	0.95	1.04	0.51
$\gamma = 5$	4.47	5.65	7.40	10.22	3.60	4.73	6.43	9.15	0.84	0.87	0.92	0.98	0.64
$\gamma = 7.5$	5.28	6.81	9.23	13.55	4.38	5.87	8.23	12.47	0.86	0.89	0.92	0.96	0.72
$\beta = 0.96$													
	λ^T				λ^B				λ^R				λ
$\hat{\theta}_2$	0.54	0.58	0.63	0.68	0.54	0.58	0.63	0.68	0.54	0.58	0.63	0.68	-
$\gamma = 1$	2.63	3.86	4.86	5.62	1.58	2.90	3.92	4.67	1.03	0.94	0.90	0.91	0.40
$\gamma = 2.5$	3.25	4.81	6.16	7.28	2.09	3.77	5.16	6.29	1.14	1.00	0.94	0.94	0.58
$\gamma = 5$	4.14	6.18	8.10	9.93	2.81	5.04	7.04	8.87	1.29	1.08	0.99	0.97	0.69
$\gamma = 7.5$	5.44	8.24	11.25	14.78	3.91	6.97	10.09	13.62	1.47	1.19	1.05	1.02	0.76
$\beta = 0.97$													
	λ^T				λ^B				λ^R				λ
$\hat{\theta}_2$	0.54	0.58	0.63	0.68	0.54	0.58	0.63	0.68	0.54	0.58	0.63	0.68	-
$\gamma = 1$	3.52	4.36	5.55	7.31	2.12	2.79	3.77	5.24	1.37	1.53	1.72	1.97	0.54
$\gamma = 2.5$	4.51	5.63	7.25	9.71	3.38	4.42	5.92	8.22	1.09	1.16	1.26	1.38	0.67
$\gamma = 5$	5.32	6.76	8.94	12.51	4.30	5.68	7.78	11.24	0.98	1.02	1.08	1.14	0.75
$\gamma = 7.5$	6.02	7.82	10.75	16.26	5.01	6.77	9.62	15.02	0.96	0.99	1.03	1.08	0.80

Notes: The table displays the computed parameters for the decomposition of the welfare cost of total economic fluctuations using a robustness sample with only the original Barro and Ursúa (2010) data. The numbers are obtained using equations (10) through (15) with the estimates shown in Table A.9. All of the entries are in percentages of lifetime consumption. We also report an extra welfare cost measure, λ, described in Appendix A.4. We report numbers for the relative degree of risk aversion $\gamma \in \{1, 2.5, 5.10\}$, for the implied $\hat{\theta}_2$ along the grid for $\theta_1 \in \{0, 0.1, 0.2, 0.3\}$, and with $\beta \in \{0.95, 0.96, 0.97\}$ for the permanent shocks.

Table A.11: Barro and Ursúa (2008) Data - Welfare Cost - ARIMA(1,1,0)

$\beta = 0.95$													
	λ^T				λ^B				λ^R				λ
$\hat{\theta}_2$	0.62	0.66	0.70	0.74	0.62	0.66	0.70	0.74	0.62	0.66	0.70	0.74	-
$\gamma = 1$	3.81	4.73	6.02	7.93	2.36	3.10	4.16	5.78	1.42	1.58	1.78	2.04	0.54
$\gamma = 2.5$	5.39	6.76	8.75	11.82	4.13	5.39	7.24	10.12	1.21	1.30	1.41	1.55	0.73
$\gamma = 5$	6.69	8.62	11.65	16.96	5.52	7.38	10.31	15.47	1.11	1.16	1.22	1.30	0.83
$\gamma = 7.5$	7.84	10.56	15.48	27.76	6.70	9.35	14.18	26.25	1.07	1.10	1.15	1.20	0.88
$\beta = 0.96$													
	λ^T				λ^B				λ^R				λ
$\hat{\theta}_2$	0.62	0.66	0.70	0.74	0.62	0.66	0.70	0.74	0.62	0.66	0.70	0.74	-
$\gamma = 1$	4.82	5.98	7.63	10.09	2.97	3.91	5.27	7.32	1.79	2.00	2.25	2.57	0.67
$\gamma = 2.5$	6.13	7.70	9.99	13.56	4.70	6.14	8.27	11.62	1.37	1.47	1.59	1.74	0.82
$\gamma = 5$	7.24	9.36	12.73	18.77	5.98	8.03	11.28	17.15	1.19	1.24	1.30	1.39	0.89
$\gamma = 7.5$	8.34	11.31	16.84	31.84	7.14	10.04	15.45	30.21	1.12	1.16	1.20	1.26	0.92
$\beta = 0.97$													
	λ^T				λ^B				λ^R				λ
$\hat{\theta}_2$	0.62	0.66	0.70	0.74	0.62	0.66	0.70	0.74	0.62	0.66	0.70	0.74	-
$\gamma = 1$	6.52	8.11	10.38	13.77	4.01	5.28	7.13	9.95	2.42	2.69	3.03	3.47	0.91
$\gamma = 2.5$	7.08	8.92	11.62	15.88	5.43	7.12	9.63	13.61	1.57	1.68	1.82	2.00	0.94
$\gamma = 5$	7.88	10.24	14.03	21.00	6.52	8.79	12.45	19.22	1.27	1.33	1.40	1.49	0.96
$\gamma = 7.5$	8.91	12.19	18.46	37.69	7.64	10.84	16.99	35.89	1.18	1.22	1.26	1.32	0.97

Notes: The table displays the computed parameters for the decomposition of the welfare cost of total economic fluctuations using a robustness sample with only the original Barro and Ursúa (2010) data. The numbers are obtained using equations (10) through (15) with the estimates shown in Table A.9. All of the entries are in percentages of lifetime consumption. We also report an extra welfare cost measure, λ, described in Appendix A.4. We report numbers for the relative degree of risk aversion $\gamma \in \{1, 2.5, 5.10\}$, for the implied $\hat{\theta}_2$ along the grid for $\theta_1 \in \{0, 0.1, 0.2, 0.3\}$, and with $\beta \in \{0.95, 0.96, 0.97\}$ for the permanent shocks.

Table A.12: Barro and Ursúa (2008) Data - Welfare Cost - ARIMA(0,1,1)

	$\beta = 0.95$												
	λ^T				λ^B				λ^R				λ
$\hat{\theta}_2$	0.62	0.66	0.70	0.74	0.62	0.66	0.70	0.74	0.62	0.66	0.70	0.74	-
$\gamma = 1$	3.30	4.09	5.21	6.85	2.04	2.68	3.61	5.00	1.23	1.37	1.54	1.77	0.46
$\gamma = 2.5$	4.70	5.87	7.57	10.16	3.59	4.68	6.26	8.69	1.06	1.14	1.23	1.35	0.64
$\gamma = 5$	5.83	7.45	9.92	14.09	4.79	6.35	8.74	12.78	0.99	1.03	1.09	1.16	0.74
$\gamma = 7.5$	6.76	8.91	12.54	20.01	5.74	7.83	11.38	18.72	0.97	1.00	1.04	1.09	0.80
	$\beta = 0.96$												
	λ^T				λ^B				λ^R				λ
$\hat{\theta}_2$	0.62	0.66	0.70	0.74	0.62	0.66	0.70	0.74	0.62	0.66	0.70	0.74	-
$\gamma = 1$	4.16	5.16	6.57	8.67	2.57	3.38	4.54	6.31	1.55	1.72	1.94	2.22	0.58
$\gamma = 2.5$	5.31	6.65	8.60	11.60	4.07	5.31	7.11	9.92	1.20	1.28	1.39	1.52	0.72
$\gamma = 5$	6.27	8.04	10.76	15.42	5.16	6.86	9.49	14.01	1.05	1.10	1.16	1.23	0.79
$\gamma = 7.5$	7.15	9.47	13.45	21.98	6.08	8.34	12.23	20.61	1.01	1.05	1.09	1.14	0.83
	$\beta = 0.97$												
	λ^T				λ^B				λ^R				λ
$\hat{\theta}_2$	0.62	0.66	0.70	0.74	0.62	0.66	0.70	0.74	0.62	0.66	0.70	0.74	-
$\gamma = 1$	5.61	6.96	8.90	11.77	3.45	4.55	6.13	8.53	2.08	2.31	2.61	2.98	0.78
$\gamma = 2.5$	6.11	7.67	9.95	13.48	4.68	6.12	8.23	11.54	1.37	1.46	1.58	1.74	0.82
$\gamma = 5$	6.79	8.73	11.76	17.02	5.60	7.46	10.39	15.49	1.13	1.18	1.24	1.32	0.85
$\gamma = 7.5$	7.59	10.11	14.50	24.43	6.46	8.91	13.21	22.96	1.06	1.10	1.14	1.19	0.87

Notes: The table displays the computed parameters for the decomposition of the welfare cost of total economic fluctuations using a robustness sample with only the original Barro and Ursúa (2010) data. The numbers are obtained using equations (18) through (15) with the estimates shown in Table A.9. All of the entries are in percentages of lifetime consumption. We also report an extra welfare cost measure, λ, described in Appendix A.4. We report numbers for the relative degree of risk aversion $\gamma \in \{1, 2.5, 5.10\}$, for the implied $\hat{\theta}_2$ along the grid for $\theta_1 \in \{0, 0.1, 0.2, 0.3\}$, and with $\beta \in \{0.95, 0.96, 0.97\}$ for the permanent shocks.

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