The Welfare Costs of Business Cycles Unveiled: Measuring the Extent of Stabilization Policies*

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Abstract

How can we measure the welfare benefit of ongoing stabilization policies? We develop a methodology to calculate the welfare cost of business cycles taking into account that observed consumption is partially smoothed. We propose a decomposition that disentangles consumption in a mix of laissez-faire (absent policies) and riskless components. With a novel identification strategy, we estimate the span of stabilization power. In our preferred specification, we find that the welfare cost of total fluctuations is 11 percent of lifetime consumption, of which 82 percent is smoothed by the status quo policies, yielding a residual 1.8 percent of consumption to be tackled by policymakers.

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JEL Classifications: E32, E21, E63, N10

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1 Introduction

In a tough challenge to conventional wisdom, Lucas (1987) asked how much Americans would be willing to pay, in terms of consumption, to live in an economy that is not subject to the macroeconomic volatility that the US witnessed during the post-war period. Finding that a representative consumer would sacrifice at most one-tenth of a percent of lifetime consumption, Lucas concluded that there would be little benefit in further attempting to stabilize the residual risk of business cycles.

Not surprisingly, Lucas's seminal result attracted a great deal of controversy and generated a wealth of literature that revisits his estimates. In this paper, we explore a critical point, which is subtly present in Lucas (1987), that calls for a new measurement effort when estimating the costs of business cycles: all observed consumption is already partially smoothed. That is, the data that we gather for consumption stem from a realized allocation that is subject to the status quo of economic stabilization policies.

In order to measure the contribution of ongoing policies as well as the relevance of the residual to be smoothed, we then need to disentangle which part of the observed consumption pertains to each category. To accomplish such a task, we propose a tractable decomposition in which observed consumption is a weighted geometric mean of laissez-faire consumption, i.e., the counterfactual consumption series in the absence of any policy and a riskless consumption sequence.

Our decomposition allows us to map all policies to a single parameter θ, which we define as the span of stabilization power. Within this structure, we are able to prove that the welfare cost of total economic fluctuations can be disentangled into the benefit of ongoing policies and the cost of residual fluctuations. We dialogue directly with the classic literature and use the flexibility of this approach to apply our formulation to three types of shock structures for the consumption process: the one of Lucas (1987) with transitory shocks, the one of Obstfeld (1994) with permanent innovations, and a third one that departs from the i.i.d. structure and uses an ARIMA process for the consumption series as proposed by Reis (2009), which we are able to incorporate into our framework with the use of the Beveridge-Nelson decomposition (Beveridge and Nelson, 1981; Issler et al., 2008; Guillén et al., 2014).

We then proceed to estimate the parameters in our welfare decomposition but hit a measurement challenge: since the laissez-faire consumption is not observable, we need to identify θ. For this task, we resort to the more novel literature of identification in macroeconomics and couple it with the relevant facts of US macroeconomic history. Our choice of data is an augmented version of the historical consumption series provided by Barro and Ursúa (2010), which shows a significant decrease in volatility after WWII. Such a pattern is identified and confirmed by (i) the established literature on the topic; (ii) the visual inspection of the data; and (iii) a statistical test (ICSS) that finds structural breaks in the variance of time series, which points to 1947 as the only observation in our sample when such a break occurs.

These three pieces of evidence allow us to design our identification strategy: we divide the sample into pre- and post-war periods with distinct measured volatilities, and thus two θ's, attributing them to the larger role and presence of stabilization policies in the second period. We then assume that the laissez-faire consumption volatility remains unchanged during the whole sample and that the span of stabilization policies in the first period, θ_1, can be considered as given at a low level due to the incipient presence of stabilization policies in the pre-war period.[1] Such a discontinuity-based strategy enables us to pin down the span of stabilization policies from 1947 until today, i.e. θ_2, which we can then estimate and use as an input in our decomposition of the welfare costs of business cycles.

Assuming a log-normal form for consumption, we obtain the results for all the three aforementioned shock structures, but our preferred specification is the one stemming from the ARIMA process, which, among three considered, best models and fits the time series of consumption. The first difference of the series follows an AR(1) process after 1947, making it straightforward to use the Beveridge-Nelson decomposition to obtain our estimates. We find that the span of stabilization policies, $\hat{\theta}_2$, smooths 61 to 73 percent

[1] In our empirical approach we consider a set with distinct possible values for θ_1 defined at a chosen grid.

of the laissez-faire consumption shocks in the post-war period.

Our identification strategy and statistical testing essentially reduce structural changes in the economy after 1947 to a unique change of value in θ that remains constant until the final period of the sample. There are several potential explanations for the observed reduction in post-war consumption volatility that could lie beyond the overarching umbrella of a unique paradigm shift of stabilization policies. In order to consider such alternative explanations and allow more flexibility to our approach, we also estimate a time-varying θ. Inspired by Stock and Watson (2007), we use an analogous approach to their exercise on changes in the post-war univariate inflation process and estimate a process with stochastic volatility for our consumption series after 1947. With this methodology, we are able to recover a stochastic $\hat{\theta}_{2,t}$ for the period, relaxing part of our identification strategy. We find that the estimated time-varying span smooths consumption in a range that gravitates close to our initial estimate for the whole post-1947 period, with its being being lower than 76 percent.

Given the closeness of the time-varying θ estimates to the values obtained in our initial two-period approach, we are able to return to them and select one of the initially estimated values for $\hat{\theta}_2$. This then allows us to use our theoretical decomposition, plug in the estimated values, and compute the different welfare costs. We find the total cost of economic fluctuations to be 11 percent of lifetime consumption. Close to 82 percent of such costs are already covered by stabilization policies, yielding that more than 9 percent of the smoothed lifetime consumption is left unveiled if one does not take into account the benefit of ongoing stabilization policies. Since the residual 1.8 percent of the costs still to be smoothed is the easiest measure to compare with the value that would be implied by the literature in our framework, we are able to find a residual cost that is two times higher than the usual numbers even when taking into account that observed consumption is partially smoothed.

In order to check the robustness of our analyses, we tackle the possibility that the log-consumption series has a structural break that we should consider beyond the one identified in its volatility. We conduct a Bai-Perron test (Bai and Perron, 2003) and find that there is one break in the first difference of log-consumption in 1934. We adjust the sample, run the same regression, and find a decrease of only 1 percentage point in $\hat{\theta}_2$, reinforcing our initial findings.[2]

Roadmap. The paper is organized as follows. Section 2 reviews the literature and discusses our contribution. Section 3 describes the model and lays out our theoretical results. Section 4 applies the results of the previous sections to three different applications. Section 5 outlines our empirical approach and describes our identification strategy. Section 6 shows our estimation results and an exercise with a time-variant θ. Section 7 uses the estimates and shows the computed results for welfare costs. Section 8 discusses a robustness exercise on structural breaks. Finally, Section 9 concludes the paper.

2 Related Literature

Our paper is embedded in three major strands of the literature in macroeconomics: (i) the large body of work concerned with the calculus of the welfare costs of business cycles; (ii) the literature that studies the measurement of historical macroeconomic data; and (iii) the literature of identification in macroeconomics.

Several papers build on Lucas's departing point and relax some of his assumptions. For example, Obstfeld (1994) switches the original transitory shocks for permanent ones and focuses on the interaction with recursive preferences; Reis (2009) further develops the time-series aspects, while Issler et al. (2008) and Guillén et al. (2014) combine both types of shocks.[3] Another block in this body departs from the representative agent setting and estimates the costs under incomplete markets and heterogeneous agents such as in İmrohoroğlu (1989), Krusell and Smith Jr. (1999), Storesletten et al. (2001), and De Santis (2007). More recently, Hai et al. (2020) include memorable goods[4] and Constantinides

[2]We conduct further robustness analyses on the estimation procedure of θ_2. In Appendix F, we adjust the sample used in the regression with the removal of the inter-war period and also with the original data sample by Barro and Ursúa (2010). The results are similar and consistent with our main analysis.

[3]For an in-depth early discussion of this literature, see Barlevy (2005), who discusses other seminal references such as Dolmas (1998) and Alvarez and Jermann (2004).

[4]A good, as defined in Hai et al. (2020), is "memorable if a consumer draws utility from her past con-

(2021) focuses on the role of idiosyncratic shocks faced by households that are unrelated to the business cycle. Our contribution here is twofold: first, we bring attention to the fact that the empirically observed consumption series is a partially smoothed series and connect it to its potential consequences for the calculation of welfare costs; second, we propose and compute a new and tractable decomposition that allows us to disentangle and reveal the reach of the ongoing stabilization policies.

We conduct our data analysis grounding it in the literature on macroeconomic history. Our sample is built directly from the historical data compiled by Barro and Ursúa (2010) and when developing our novel identification strategy, we base it on Barro and Ursúa (2008)'s observation that for the OECD economies, there is a change in consumption volatility in the post-war period. Our approach also dialogues with the seminal work of Romer (1986) and Balke and Gordon (1989) that documents the challenges faced when measuring the volatility of macroeconomic aggregates, and we show how our methodology can reconcile improvements in both measurement and stabilization after WWII. Here we add our estimation of the unique structural break in the volatility of consumption in 1947 as measured by the Inclan and Tiao (1994) test that is used in our identification exercise.

We also view our work as building on the effort of calculating the costs of business cycles, with critical attention to measurement and identification that often appeared in what became known as the "disasters" approach in the literature. We resort to Nakamura et al. (2013)'s insight of using the variation in the volatility of the consumption series to better identify the shift in the role of stabilization policies. Moreover, we build on Nakamura et al. (2017) in our use of both transitory and permanent formulations for the shocks in conjunction with a time-varying volatility for the consumption series. Our paper contributes here by using different methodologies to model the measured time-series aspects of the consumption data. For example, we use the Beveridge-Nelson decomposition to tie back our methodology to its ARIMA components and also, to the extent of our knowledge, we are the first to connect the Stock and Watson (2007) methodology for the inflation

sumption experience."

process to its time-varying volatility.

Since we find large values for our estimates of welfare costs, we are also connected with the intersection of the disasters and welfare costs literature. For instance, Jorda et al. (2020) find that substantial costs may arise from a novel estimate of frequent and small disasters.[5] In addition, by considering the asymmetric nature of economic fluctuations, Dupraz et al. (2019) develop a plucking model of business cycles and find welfare gains from eliminating economic fluctuations that are an order of magnitude larger than in the standard models.

3 Model

3.1 Environment and Definitions

The economy is populated by a representative consumer whose lifetime utility is given by $\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t u(C_t)\right]$, where C_t is consumption in period t, $\beta \in (0,1)$ is an intertemporal discount factor, $u(\cdot)$ is the instantaneous utility function, and $\mathbb{E}_0[\cdot]$ is the expectation operator conditional on the information set $\mathcal{I}_0$.[6] We begin with a few definitions:

Definition 1. *Define $\bar{C}_t \equiv \mathbb{E}_0[C_t]$. Then $\{\bar{C}_t\}_{t=0}^{\infty}$ is the riskless consumption sequence.*

Definition 2. *Define $\tilde{C}_t$ as consumption in the absence of stabilization policies. Then $\{\tilde{C}_t\}_{t=0}^{\infty}$ is the laissez-faire consumption sequence.*

We can now define the welfare cost of the total economic fluctuations as the constant $\lambda^T > 0$ that solves the following condition:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t u\left((1+\lambda^T)\tilde{C}_t\right)\right] = \sum_{t=0}^{\infty}\beta^t u\left(\bar{C}_t\right). \tag{1}$$

[5]Other examples in this literature are Barro and Jin (2011) and Gourio (2012).

[6]We assume that the expectation is taken before the realization of any uncertainty in period 0, as in some calculations done by Obstfeld (1994) and Reis (2009). In that sense, consumption in that period is treated as a stochastic variable. Under this assumption we compare the expected utility in two worlds where the agent is still uncertain about all consumption flows, as in Lucas (1987).

The parameter λ^T measures the constant compensation required by the consumer to be indifferent between the adjusted laissez-faire, $\{(1+\lambda^T)\tilde{C}_t\}_{t=0}^{\infty}$, and the riskless consumption sequences.

Given that the observed time series on consumption is subject to the ongoing stabilization policies, we can view it as the combination of two extreme cases: (i) the (non-observed) consumption series in the absence of any stabilization policies, $\tilde{C}_t$, and (ii) the (non-observed) perfectly smoothed consumption, $\bar{C}_t$. We then model the (observed) partially smoothed consumption as a weighted geometric average:

$$C_t(\theta) \equiv \bar{C}_t^{\theta} \tilde{C}_t^{1-\theta}, \tag{2}$$

where the parameter $\theta \in [0,1]$ measures the degree of consumption smoothing. Thus, θ can be interpreted as the span of the stabilization power of governmental policies.

We can now define the benefit of the ongoing stabilization policies as the constant $\lambda^B > 0$ that solves the following condition:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u\left((1+\lambda^B)\tilde{C}_t \right) \right] = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(C_t(\theta)) \right]. \tag{3}$$

The parameter λ^B is the compensation required by the consumer to be indifferent between the adjusted laissez-faire consumption sequence and the effective consumption sequence, $\{C_t(\theta)\}_{t=0}^{\infty}$.

Finally, we can compute what is left to be stabilized by defining the welfare cost of the residual economic fluctuations as the constant $\lambda^R > 0$ that solves the following condition:

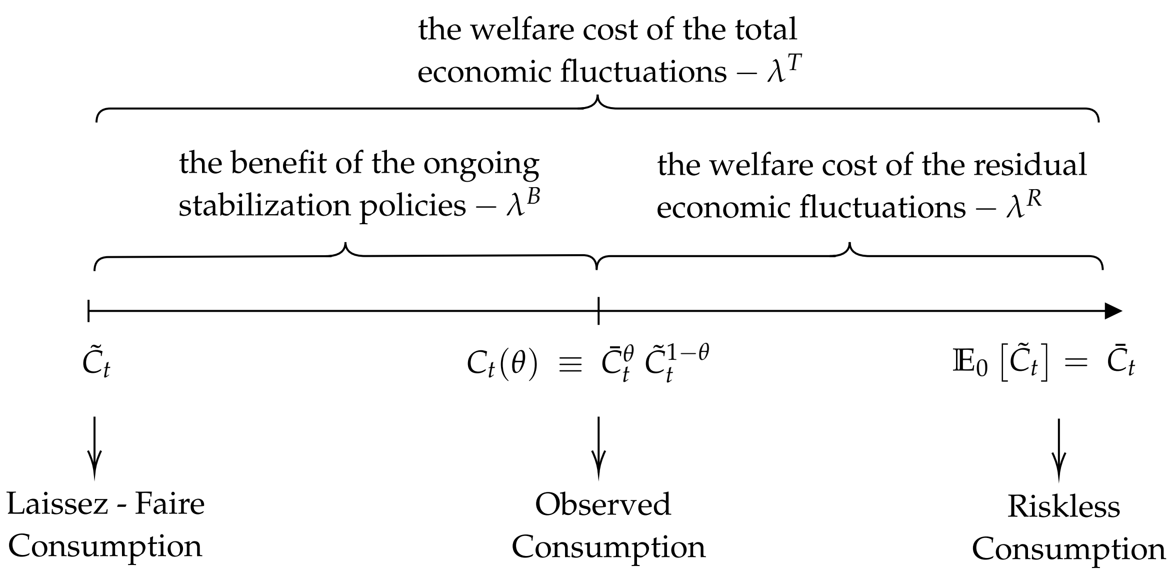
$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u\left((1+\lambda^R)C_t(\theta) \right) \right] = \sum_{t=0}^{\infty} \beta^t u\left(\bar{C}_t\right). \tag{4}$$

The parameter λ^R measures the constant compensation required by the consumer to be indifferent between the adjusted partially smoothed consumption sequence $\{(1+$

$\lambda^R)C_t(\theta)\}_{t=0}^{\infty}$ and the aforementioned riskless sequence.

Figure 1 summarizes our modelling by showing where each parameter and measure defined is located in a spectrum of consumption that spans the highest to the lowest level of risk.

Figure 1: Decomposition of the welfare cost of the total economic fluctuations



3.2 Assumptions

In order to calculate λ^T, λ^B, and λ^R and guarantee tractability, we assume a log-normal process for $\tilde{C}_t$, which implies that $C_t(\theta)$ is also log-normal. Following Lucas (1987), we assume a CRRA instantaneous utility with parameter γ:

$$u(C) = \begin{cases} \dfrac{C^{1-\gamma}}{1-\gamma}, \text{ if } \gamma > 1 \\ \ln(C), \text{ if } \gamma = 1 \end{cases} \tag{5}$$

We also need assumptions that guarantee that the sums in conditions (1), (3), and (4) are all finite. They are:

Assumption 1. *Log-normal consumption process:* $\tilde{C}_t = \alpha_0(1+\alpha_1)^t X_t$, *where* $X_t = e^{x_t - 0.5\sigma_t^2}$, *with* $x_t|\mathcal{I}_0 \sim \mathcal{N}\left(0, \sigma_t^2\right)$.

Assumption 2. *The constant* $\Gamma \equiv \beta\left(1+\alpha_1\right)^{1-\gamma} \in (0,1)$.

Assumption 3. $\sum_{t=0}^{\infty} \Gamma^t \exp\left\{-0.5\gamma\left(1-\gamma\right)\sigma_t^2\right\} < \infty$.[7]

Under Assumption 1, riskless consumption is given by $\bar{C}_t = \mathbb{E}_0[\tilde{C}_t] = \alpha_0(1+\alpha_1)^t$ and is deterministic. Furthermore, $\tilde{C}_t = \bar{C}_t X_t$, and the partially smoothed consumption can be rewritten as $C_t\left(\theta\right) = \bar{C}_t X_t^{1-\theta}$. From this formulation it is easy to see that the larger the parameter θ, the less important is the stochastic part of the partially smoothed consumption.

3.3 Theoretical Results

We can now derive closed-form solutions for the parameters λ^B, λ^R and λ^T. Propositions 1, 2, and 3 establish, respectively, each of these parameters. The final step consists of using the propositions to obtain our main decomposition of the welfare cost of total economic fluctuations. All proofs are shown in Appendix A.

Proposition 1. *Under Assumptions 1 and 3 the benefit of the ongoing stabilization policies is given by*

$$\lambda^B = \begin{cases} \exp\left\{\theta \frac{1-\beta}{2} \sum_{t=0}^{\infty} \beta^t \sigma_t^2\right\} - 1, & \text{if } \gamma = 1 \\ \left[\frac{\sum_{t=0}^{\infty} \Gamma^t \exp\left\{-0.5(1-\gamma)(1-\theta)(\theta+\gamma-\gamma\theta)\sigma_t^2\right\}}{\sum_{t=0}^{\infty} \Gamma^t \exp\left\{-0.5\gamma(1-\gamma)\sigma_t^2\right\}}\right]^{\frac{1}{1-\gamma}} - 1, & \text{if } \gamma > 1 \end{cases} \tag{6}$$

Proposition 2. *Under Assumptions 1, 2, and 3 the welfare cost of the residual macroeconomic fluctuations is given by*

$$\lambda^R = \begin{cases} \exp\left\{(1-\theta) \frac{1-\beta}{2} \sum_{t=0}^{\infty} \beta^t \sigma_t^2\right\} - 1, & \text{if } \gamma = 1 \\ \left[\frac{\sum_{t=0}^{\infty} \Gamma^t}{\sum_{t=0}^{\infty} \Gamma^t \exp\left\{-0.5(1-\gamma)(1-\theta)(\theta+\gamma-\gamma\theta)\sigma_t^2\right\}}\right]^{\frac{1}{1-\gamma}} - 1, & \text{if } \gamma > 1 \end{cases} \tag{7}$$

[7] Note that $\sum_{t=0}^{\infty} \Gamma^t \exp\left\{-0.5\left(1-\theta\right)\left(1-\gamma\right)\left(\gamma+\theta-\gamma\theta\right)\sigma_t^2\right\} < \sum_{t=0}^{\infty} \Gamma^t \exp\left\{-0.5\gamma\left(1-\gamma\right)\sigma_t^2\right\}$ if $\gamma > 1$. This result ensures that the λ's are finite in some of our results.

Proposition 3. *Under Assumptions 1, 2, and 3 the welfare cost of the total economic fluctuations is given by*

$$\lambda^T = \begin{cases} \exp\left\{\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\sigma_t^2\right\} - 1, & \text{if } \gamma = 1 \\ \left[\frac{\sum_{t=0}^{\infty}\Gamma^t}{\sum_{t=0}^{\infty}\Gamma^t \exp\left\{-0.5\gamma(1-\gamma)\sigma_t^2\right\}}\right]^{\frac{1}{1-\gamma}} - 1, & \text{if } \gamma > 1 \end{cases} \tag{8}$$

We can now state our main result in Theorem 1 below: the decomposition of the welfare cost of total economic fluctuations.

Theorem 1. *Under Assumptions 1 to 3 and CRRA utility (5), there is a decomposition of the welfare cost of total economic fluctuations in the form*

$$1 + \lambda^T = \left(1 + \lambda^B\right)\left(1 + \lambda^R\right). \tag{9}$$

4 Applications

In this section we characterize λ^T, λ^B, and λ^R using three different shock structures for the consumption process: the classic ones of Lucas (1987) with transitory shocks and of Obstfeld (1994) with permanent shocks, and one with an ARIMA process for consumption as proposed in Reis (2009) using the Beveridge-Nelson (BN) decomposition (Beveridge and Nelson, 1981; Issler et al., 2008; Guillén et al., 2014). The details of all calculations are shown in Appendix B.

Example 1 - Transitory Shocks (Lucas, 1987): Define $C_t = \alpha_0(1+\alpha_1)^t e^{-0.5\sigma_\varepsilon^2 + x_t^L}$, where $x_t^L|\mathcal{I}_0 \sim \mathcal{N}(0, \sigma_\varepsilon^2)$. Hence,

$$\lambda^T = \begin{cases} \exp\left\{\frac{1}{2}\sigma_\varepsilon^2\right\} - 1, & \text{if } \gamma = 1 \\ \exp\left\{\frac{\gamma}{2}\sigma_\varepsilon^2\right\} - 1, & \text{if } \gamma > 1 \end{cases} \tag{10}$$

$$\lambda^B = \begin{cases} \exp\left\{\frac{\theta}{2}\sigma_\varepsilon^2\right\} - 1, & \text{if } \gamma = 1 \\ \exp\left\{\frac{\gamma}{2}\sigma_\varepsilon^2 - \frac{1}{2}(1-\theta)(\theta+\gamma-\gamma\theta)\sigma_\varepsilon^2\right\} - 1, & \text{if } \gamma > 1 \end{cases} \tag{11}$$

$$\lambda^R = \begin{cases} \exp\left\{\frac{1-\theta}{2}\sigma_\varepsilon^2\right\} - 1, & \text{if } \gamma = 1 \\ \exp\left\{\frac{1}{2}(1-\theta)(\theta+\gamma-\gamma\theta)\sigma_\varepsilon^2\right\} - 1, & \text{if } \gamma > 1 \end{cases} \tag{12}$$

For this process, the variance in Assumption 1 becomes $\sigma_t^2 = \sigma_\varepsilon^2$. Consequently, Assumption 3 is satisfied as long as Assumption 2 holds.

Example 2 - Permanent Shocks (Obstfeld, 1994): Define $C_t = \alpha_0(1+\alpha_1)^t e^{-0.5\sigma_\varepsilon^2 + x_t^O}$, where $x_t^O = \sum_{i=0}^t \varepsilon_i$, $\varepsilon_i|\mathcal{I}_0 \sim \mathcal{N}(0, \sigma_\varepsilon^2)$.[8] Thus,

$$\lambda^T = \begin{cases} \exp\left\{\frac{1}{2}\frac{1}{1-\beta}\sigma_\varepsilon^2\right\} - 1, & \text{if } \gamma = 1 \\ \exp\left\{0.5\gamma\sigma_\varepsilon^2\right\}\left[\frac{1-\Gamma\exp\left\{-0.5\gamma(1-\gamma)\sigma_\varepsilon^2\right\}}{1-\Gamma}\right]^{\frac{1}{1-\gamma}} - 1, & \text{if } \gamma > 1 \end{cases} \tag{13}$$

$$\lambda^B = \begin{cases} \exp\left\{\theta\frac{1}{2}\frac{1}{1-\beta}\sigma_\varepsilon^2\right\} - 1, & \text{if } \gamma = 1 \\ \frac{\exp\left\{0.5\gamma\sigma_\varepsilon^2\right\}}{\exp\left\{0.5(1-\theta)[\gamma+\theta-\theta\gamma]\sigma_\varepsilon^2\right\}}\left[\frac{1-\Gamma\exp\left\{-0.5\gamma(1-\gamma)\sigma_\varepsilon^2\right\}}{1-\Gamma\exp\left\{-0.5(1-\gamma)(1-\theta)(\theta+\gamma-\gamma\theta)\sigma_\varepsilon^2\right\}}\right]^{\frac{1}{1-\gamma}} - 1, & \text{if } \gamma > 1 \end{cases} \tag{14}$$

$$\lambda^R = \begin{cases} \exp\left\{(1-\theta)\frac{1}{2}\frac{1}{1-\beta}\sigma_\varepsilon^2\right\} - 1, & \text{if } \gamma = 1 \\ \frac{1}{\exp\left\{0.5(1-\theta)[\gamma+\theta-\theta\gamma]\sigma_\varepsilon^2\right\}}\left[\frac{1-\Gamma\exp\left\{-0.5(1-\gamma)(1-\theta)(\theta+\gamma-\gamma\theta)\sigma_\varepsilon^2\right\}}{1-\Gamma}\right]^{\frac{1}{1-\gamma}} - 1, & \text{if } \gamma > 1 \end{cases} \tag{15}$$

In this case, $\sigma_t^2 = Var_0\left[\sum_{i=0}^t \varepsilon_i\right] = (t+1)\sigma_\varepsilon^2$, and the condition $\Gamma\exp\{-0.5\gamma(1-\gamma)\sigma_\varepsilon^2\} <$

[8] In some calculations, Obstfeld (1994) treats C_0 as known. We consider the case where the expectation is taken before the realization of the shock ε_0.

1 is sufficient for Assumption 3 to be valid.

Example 3 - ARIMA-BN Process (Reis, 2009): Define

$$C_t = \alpha_0(1+\alpha_1)^t \exp\left\{-\frac{1}{2}\sigma^2_{x_t^{BN}}\right\} \exp\left\{x_t^{BN}\right\} \tag{16}$$

where, to obtain x_t^{BN}, we apply the Beveridge-Nelson decomposition. We follow these steps:

1. Given a process, $C_t = f(t) + u_t$, where $f(t)$ is deterministic and $(1-L)u_t = \psi(L)\varepsilon_t$, with $\psi(L) = \sum_{j=0}^{\infty} \psi_j L^j$. Define $\varphi_j = -\sum_{i=j+1}^{\infty} \psi_i$.
2. Then, $x_t^{BN} = \psi(1) \sum_{j=0}^{t} \varepsilon_j + \sum_{j=0}^{t} \varphi_j \varepsilon_{t-j}$, with $\varepsilon_j|\mathcal{I}_0 \sim \mathcal{N}(0, \sigma^2_\varepsilon)$.
3. We follow Issler et al. (2008) and rewrite $\sigma^2_{x_t^{BN}}$ as $\tilde{\sigma}^2_{x_t^{BN}} = \rho_0 + \rho_1 t$, where

$$\rho_0 \equiv \psi(1)^2 \sigma^2_\varepsilon + 2\psi(1) \sum_{j=0}^{\infty} \varphi_{t-j}\sigma^2_\varepsilon + \sum_{j=0}^{\infty} \varphi^2_{t-j}\sigma^2_\varepsilon \quad \text{and} \quad \rho_1 \equiv \psi(1)^2 \sigma^2_\varepsilon \tag{17}$$

4. Hence, since we know $x_t^{BN} \sim N\left(0, \sigma^2_{x_t^{BN}}\right)$ and can approximate $\sigma^2_{x_t^{BN}}$ with $\tilde{\sigma}^2_{x_t^{BN}}$.

We can then compute the λ's for this structure of shocks:[9]

[9] See some omitted calculations for the steps above and the characterization of the λ's in Appendix B.3

$$\lambda^T = \begin{cases} \exp\left\{\frac{1}{2}\left(\rho_0 + \frac{\beta}{1-\beta}\rho_1\right)\right\} - 1, & \text{if } \gamma = 1 \\ \exp\{0.5\gamma\rho_0\}\left[\frac{1-\Gamma\exp\{-0.5\gamma(1-\gamma)\rho_1\}}{1-\Gamma}\right]^{\frac{1}{1-\gamma}}, & \text{if } \gamma > 1 \end{cases} \tag{18}$$

$$\lambda^B = \begin{cases} \exp\left\{\frac{\theta}{2}\left(\rho_0 + \frac{\beta}{1-\beta}\rho_1\right)\right\} - 1, & \text{if } \gamma = 1 \\ \frac{\exp\{0.5\gamma\rho_0\}}{\exp\{0.5(1-\theta)(\theta+\gamma-\gamma\theta)\rho_0\}}\times & \\ \quad \times\left[\frac{1-\Gamma\exp\{-0.5\gamma(1-\gamma)\rho_1\}}{1-\Gamma\exp\{-0.5(1-\gamma)(1-\theta)(\theta+\gamma-\gamma\theta)\rho_1\}}\right]^{\frac{1}{1-\gamma}} - 1, & \text{if } \gamma > 1 \end{cases} \tag{19}$$

$$\lambda^R = \begin{cases} \exp\left\{\frac{1-\theta}{2}\left(\rho_0 + \frac{\beta}{1-\beta}\rho_1\right)\right\} - 1, & \text{if } \gamma = 1 \\ \exp\{0.5\,(1-\theta)\,(\theta+\gamma-\gamma\theta)\,\rho_0\}\times & \\ \quad \times\left[\frac{1-\Gamma\exp\{-0.5(1-\gamma)(1-\theta)(\theta+\gamma-\gamma\theta)\rho_1\}}{1-\Gamma}\right]^{\frac{1}{1-\gamma}} - 1, & \text{if } \gamma > 1 \end{cases} \tag{20}$$

In this case, $\Gamma\exp\{-0.5\gamma(1-\gamma)\rho_1\} < 1$ is sufficient for Assumption 3 to be valid.

5 Empirical Approach

In order to estimate the parameters of the underlying consumption process and compute λ^T, λ^B and λ^R, we have to be specific in our assumptions about the structure of the shocks. Following the examples in the last section, we consider the three aforementioned cases: transitory, permanent, and ARMA shocks. In this section, we first develop the regressions to be estimated in the data and characterize the challenge present in the identification of the span of stabilization power, θ, as well as the other parameters of our setting. We then present algebraically and visually the strategy we implement to overcome this difficulty, allowing us to pin down the values to be used in our results.

5.1 Estimation

5.1.1 Transitory Shocks

Assuming transitory shocks under Assumption 1 and applying the logarithm to both sides of equation (2), we have that:

$$\log\left(C_t(\theta)\right) = \log\left(\alpha_0\right) - (1-\theta)0.5\sigma_\varepsilon^2 + t\log\left(1+\alpha_1\right) + (1-\theta)\varepsilon_t. \tag{21}$$

We can reinterpret (21) as a time-series regression of log-per capita consumption c_t with coefficients π_0 and π_1, and error u_t:

$$\log(c_t) = \pi_0 + \pi_1 t + u_t, \tag{22}$$

Note that an identification problem arises when we try to estimate the parameters in equation (21) since $(\alpha_0, \theta, \sigma_\varepsilon^2)$ are all simultaneously mapped to π_0. Furthermore, σ_ε^2 is scaled by $(1-\theta)$, which lies in the background of u_t. Only parameter α_1 is well-identified and can be directly inverted from the estimates since $\alpha_1 = \exp\left(\pi_1\right) - 1$.

5.1.2 Permanent Shocks

Considering the case where permanent shocks hit consumption, we have that:

$$\log\left(C_t(\theta)\right) = \log\left(\alpha_0\right) - (1-\theta)0.5t\sigma_\varepsilon^2 + t\log\left(1+\alpha_1\right) + (1-\theta)\sum_{i=0}^{t}\varepsilon_i. \tag{23}$$

Taking first differences,

$$\Delta\log\left(C_t(\theta)\right) = \log\left(1+\alpha_1\right) - (1-\theta)0.5\sigma_\varepsilon^2 + (1-\theta)\varepsilon_t. \tag{24}$$

We can re-write equation (24) as:

$$\Delta \log(c_t) = \pi_0 + u_t. \tag{25}$$

The same identification issue arises: $(\alpha_1, \theta, \sigma_\varepsilon^2)$ are behind π_0 with σ_ε^2 scaled by $(1-\theta)$.

5.1.3 ARIMA-BN Process

Similarly, we have that:

$$\Delta \ln C_t(\theta) = \ln(1+\alpha_1) - 0.5\rho_1 + (1-\theta)\Delta x_t^{BN} \tag{26}$$

Hence,

$$\Delta \ln C_t(\theta) = \ln(1+\alpha_1) - 0.5\psi(1)^2 \sigma_\varepsilon^2 + \psi(L)\tilde{\varepsilon}_t \tag{27}$$

where $\tilde{\varepsilon}_t \sim N\left(0, (1-\theta)^2 \sigma_\varepsilon^2\right)$.

Here we use the fact that the per capita consumption series has a unit root and its first difference is stationary.[10] Hence, we can switch to the ARMA(p,q) form:

$$\Phi(L)\Delta \ln C_t(\theta) = \Phi(1)\left[\ln(1+\alpha_1) - 0.5\psi(1)^2 \sigma_\varepsilon^2\right] + \Theta(L)\tilde{\varepsilon}_t \tag{28}$$

At this step we estimate an ARMA(p,q) with an intercept for the first difference of the observed log-consumption series. After that, we have $\Phi(L)$ and $\Theta(L)$ and invert the autoregressive lag polynomial to obtain:

$$\Delta \ln C_t(\theta) = \left[\ln(1+\alpha_1) - 0.5\psi(1)^2 \sigma_\varepsilon^2\right] + \psi(L)\tilde{\varepsilon}_t \tag{29}$$

[10] The series is I(1) as identified by the ADF, PP, KPSS, and DF-GLS tests.

Which takes us back to the format shown in the time-series regression (25), where $\psi(L) = \Theta(L)/\Phi(L)$ was obtained in the estimation process, yielding us $\phi(1)$ and $\psi(1)$. This leaves us again with the scaling factor $(1-\theta)$ in the way of the identification of the process parameters.

5.2 Identification

From our previous characterization of the identification problem, we observed that the scaling of the structural parameters by θ means that the consumption series is partially smoothed due to the ongoing stabilization policies. This means that if we knew θ (or σ_ε^2) in advance, it would be possible to recover all parameters in our consumption model by running a simple regression like the ones shown previously. Since this is not possible, we need to design an identification strategy.

Our strategy consists of exploring an observed variation in the volatility of the historical consumption series in order to identify θ. We use a combination of three pieces of evidence: (i) the empirical fact documented in the literature that per capita consumption in the US became less volatile after WWII; (ii) a visual analysis in which we plot the series and observe a potentially unique break in the graph coinciding with the post-war period; and (iii) a statistical result in which we conduct a test to find any breaks in the variance series.

To apply this strategy in the data, we need to use a long series of consumption for the US. Our choice is to build on the data by Barro and Ursúa (2010). This database contains annual observations of US per capita consumption between 1834 and 2009. We complete the sequence of consumption between 2010 and 2019, maintaining their methodology and using the series available from the BEA's NIPA. Finally, we set the data in real terms to 2012.[11]

For the first factor, we follow Lucas (1987), Barro and Ursúa (2008), and Nakamura

[11] We use the series "Personal Consumption Expenditures" in Table 1.1.5, the price index series for the same category in Table 1.1.4, and the series"Population (midperiod thousands)" in Table 2.1 (US Bureau of Economic Analysis, 2021a,b,c).

et al. (2017), who discuss and document the fact that the end of the Second World War marks a substantial decrease in the volatility of consumption over time, exhibiting a heteroskedastic pattern. The second piece of evidence with the visual analysis is depicted in Figure 2. For the third factor, we apply the iterated cumulative sums of squares (ICSS) algorithm developed by Inclan and Tiao (1994) to detect breaks in the variance of consumption growth. We use a 5 percent significance level to test for multiple breaks.[12] The ICSS algorithm identifies only one break in the variance of consumption growth indicating a sudden decrease in the volatility of consumption growth after 1947. We then profit from the approach of discontinuity-based identification as discussed in Nakamura and Steinsson (2018) and assume that no other factors, aside from the changes in stabilization policies, that affect the consumption series of the US change discontinuously at the end of WWII.

In formal terms, suppose that we have two periods of time, 1 and 2, and that $Var(\varepsilon_t) = \sigma^2_\varepsilon$ in both periods, but we observe a lower volatility in consumption in period 2. All else constant, we can attribute this difference in the measured volatility to a different span of stabilization power of policies in those periods. To see that, let θ_i and $\hat{\sigma}^2_{u,i}$ be, respectively, the stabilization power and the estimated variance of u_t in period $i \in \{1, 2\}$. Thus, we have that $\hat{\sigma}^2_{u,i} = (1 - \theta_i)^2 \sigma^2_\varepsilon$. If we knew θ_1 in advance, we could pin down θ_2 using the following identifying equation:

$$\hat{\theta}_2 = 1 - (1 - \theta_1) \sqrt{\frac{\hat{\sigma}^2_{u,2}}{\hat{\sigma}^2_{u,1}}}. \tag{30}$$

The remaining parameter to delineate in the strategy is θ_1. For that, a natural candidate would be a period of incipient stabilization policies, i.e., one in which θ_1 is close to zero.

In Figure 2, we show our identification strategy at work in the plot of the historical series of consumption. The top panel 2a shows the series in its log level for the identi-

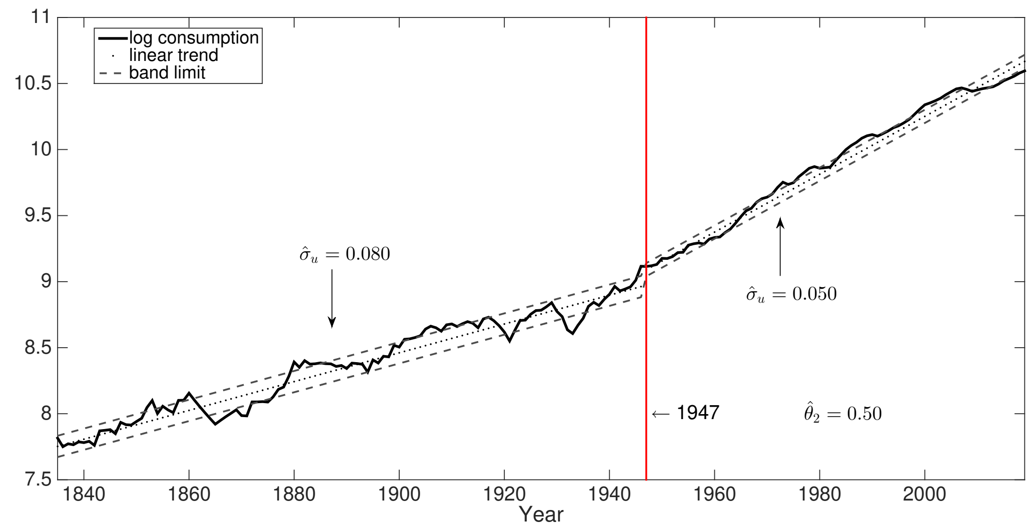
[12] We consider the critical value of 1.30 reported in Table 1 of Inclan and Tiao (1994) for a sample size of 200, which is the number closest to our sample. Considering the asymptotic value for the test (1.358) does not change our results.

fication with transitory shocks and the bottom panel 2b shows the series' first difference to accommodate the permanent shocks and ARIMA-BN process approaches as shown in equations (24) and (29).[13]

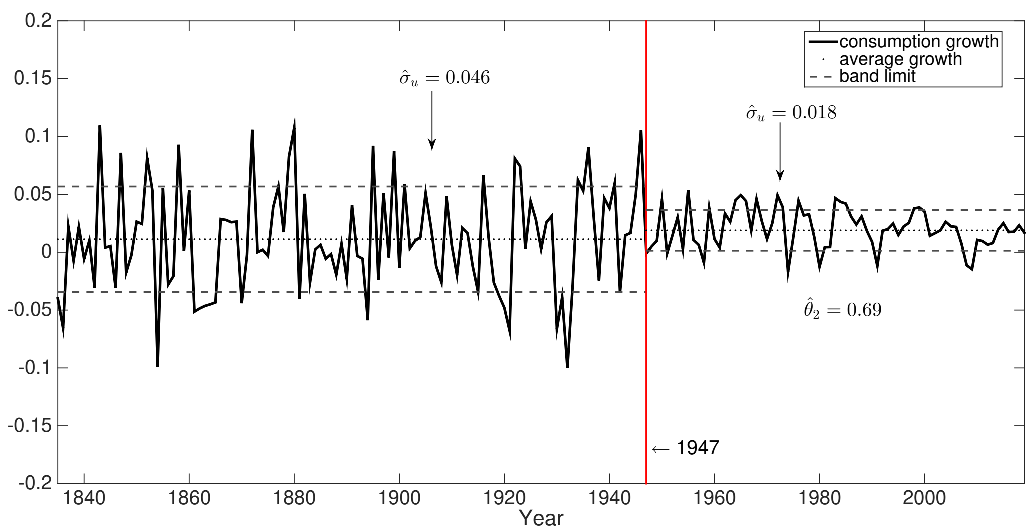
If we divide the series into two periods, pre- and post-war, there is a substantial decrease in the measured standard error after 1947. Focusing on the series with first differences in the bottom panel, for the period between 1835 and 1946, we have that $\hat{\sigma}_u = 0.046$, which then suffers a sharp decrease of more than 60 percent of its value, to $\hat{\sigma}_u = 0.018$, after WWII until today. With such a discontinuous decrease in the volatility of the series, we can plug these measures into equation (30) and, assuming $\theta_1 = 0.20$, for instance, we find that $\hat{\theta}_2 = 0.69$. This indicates a share of 69 percent of smoothed consumption in the observed series post-1947.

[13] In Appendix C, we plot in Figure 3 the visual identification for transitory consumption shocks.

Figure 2: Time series of per capita consumption for the US between 1835 and 2019.



(a) Time series of log consumption.



(b) Time series of consumption growth.

Notes: The figure shows the time series for per capita consumption for the US between 1835 and 2019 with our augmented sample of the Barro and Ursúa (2010) data. There are two panels: the top one uses the series in log levels and the second in growth. The vertical line marks the year 1947, at the end of WWII. We report the standard errors for the two sub-periods generated by this line along with the average and band limits equivalent to $2\sigma_u$. In Appendix C, we include Figure 3 showing the visual identification of the data in panel 2a in an equivalent format to the data in panel 2b.

A critical point for our measurement of the decrease in consumption's standard error is the seminal argument by Romer (1986) about the spurious decrease in the unemployment rate's volatility after 1948, which was also emphasized for GNP in Balke and Gordon (1989) and revisited for GDP in the context of OECD economies by Barro and Ursúa (2008). The first differing factor in our approach is that we use the series for consumption collected by the BEA since 1929. On top of that, with our augmented sample of Barro and Ursúa (2010) data, we add an extra 90 years to the length of Romer (1986)'s original sample.

A second relevant consideration is the fact that our methodology allows us enough flexibility for a degree of discretion in the interpretation of the span during the incipient stabilization period. In equation (30), the greater θ_1, the smaller the impact of the volatility ratio in the identification of the second period's span. In that sense, the choice of θ_1 can be made larger to reflect both a historically motivated share of riskless consumption and to also take into account a certain degree of measurement error that undermined the mapping of such stabilization to the collected data.[14]

6 Empirical Results

6.1 Estimation

We run regressions (22) and the versions of (25) for permanent shocks and ARIMA-BN process and obtain their estimated coefficients as well as the error volatility of the two distinct periods, $\hat{\sigma}^2_{u,i}$. We compute the span of stabilization power, $\hat{\theta}_2(\theta_1)$, for different levels in the grid $\theta_1 \in \{0, 0.1, 0.2, 0.3\}$, to allow different policy efficiencies in the initial period. With these values, we can then directly compute $\hat{\sigma}^2_{\varepsilon} = \hat{\sigma}^2_{u,1}/(1-\theta_1)^2$.

For the remaining parameters, in the case of transitory shocks we have that $\hat{\alpha}_1 = \exp(\hat{\pi}_1) - 1$. For permanent shocks, $\hat{\alpha}_1 = \exp\left(\hat{\pi}_0 + (1-\hat{\theta}_2)0.5\sigma^2_{\varepsilon}\right) - 1$. Finally, for the case of the ARIMA-BN process we have that $\hat{\alpha}_1 = \exp\left(\hat{\pi}_0 + (1-\hat{\theta}_2)0.5\hat{\psi}(1)^2\sigma^2_{\varepsilon}\right) - 1$. Table 1 shows the results of our estimations.

[14] Here we also develop another subtle point mentioned in Lucas's original analysis. In Lucas (1987), footnote 4, there is a mention of Romer (1986) in which the author acknowledges that his calculations do not incorporate her findings and may rely on the 1930s experience.

Table 1: Estimated parameters.

Transitory shocks				Permanent shocks				ARIMA-BN			
Estimated parameters				Estimated parameters				Estimated parameters			
	$\hat{\pi}_0$	$\hat{\pi}_1$	$\hat{\sigma}_u^2$		$\hat{\pi}_0$	$\hat{\pi}_1$	$\hat{\sigma}_u^2$		$\hat{\pi}_0$	$\hat{\phi}_1$	$\hat{\sigma}_u^2$
1835 - 1946	7.7417	0.0109	0.0065	1835 - 1946	0.0113	-	0.0021	1835 - 1946	0.0113	-	0.0021
	(.0156)	(.0002)			(.0043)				(.0043)		
1947 - 2019	6.6156	0.0219	0.0025	1947 - 2019	0.0203	-	0.0003	1947 - 2019	0.0189	0.2919	0.0003
	(.0427)	(.0003)			(.0020)				(.0030)	(.1321)	
Implied parameters				Implied parameters				Implied parameters			
θ_1	$\hat{\theta}_2$	$\hat{\alpha}_1$	$\hat{\sigma}_\varepsilon^2$	θ_1	$\hat{\theta}_2$	$\hat{\alpha}_1$	$\hat{\sigma}_\varepsilon^2$	θ_1	$\hat{\theta}_2$	$\hat{\alpha}_1$	$\hat{\sigma}_\varepsilon^2$
0.00	0.3731	0.0222	0.0065	0.00	0.6284	0.0209	0.0021	0.00	0.6142	0.0277	0.0021
0.10	0.4358	0.0222	0.0080	0.10	0.6656	0.0209	0.0026	0.10	0.6528	0.0278	0.0026
0.20	0.4985	0.0222	0.0101	0.20	0.7027	0.0210	0.0032	0.20	0.6914	0.0278	0.0032
0.30	0.5612	0.0222	0.0132	0.30	0.7399	0.0210	0.0042	0.30	0.7300	0.0279	0.0042

Notes: The table displays the estimated parameters obtained by running regression (22) for transitory shocks and regression (25) for permanent shocks and the ARIMA-BN process. The time series used is our sample of the augmented Barro and Ursúa (2010) data with sub-periods divided at 1947. The series is I(1) as identified by the ADF, PP, KPSS, and DF-GLS tests. The first difference of the series is identified, by both the AIC and BIC criteria, as an ARMA (0,0) for the pre-1947 period and an ARMA(1,0) for the subsequent years. The implied parameters are obtained using the formulas described in the text and equation (30).

Our preferred shock structure is the one with the ARIMA-BN process, since it is the one that most accurately models the data and allows for a more flexible structure without relying on the i.i.d. assumption of either the level or the first difference of the series. We also focus the discussion on the results associated with our preferred choice of initial span, $\theta_1 = 0.20$, since it allows, as mentioned previously, for a combination of some degree of stabilization power and measurement error in the pre-1947 sample. The estimated span is 0.4985 with transitory shocks, 0.7027 with permanent shocks, and 0.6814 with the ARIMA-BN process.[15]

These results show that the average post-war reach of stabilization policies is far from trivial and more than tripled after WWII. The results naturally vary according to the choice of θ_1, but with moderate sensitivity: had we considered a total absence of stabilization policies in the pre-war sample, i.e., $\theta_1 = 0$, we would have that the post-war smoothing factor would be 61 percent for the ARIMA-BN process. Moreover, for all shocks, as we increase the value of θ_1, the implied increase in $\hat{\theta}_2$ is incrementally smaller, further contributing to the robustness of the range estimated. Another feature of our strategy shown in Figure 2 is the strict division of the data in 1947. We relax the 1947 cutoff by conducting robustness checks with different windows of time in Section 8 and Appendix F.

6.2 Time-Varying θ

One critical point in our identification strategy is the sharp division of the whole time series into only two periods, pre- and post-war. More importantly, the period after 1947 exhibited a number of structural changes that fundamentally altered the US economy. Some of these could be the rise of the service industry and a sectoral change away from agriculture, skill-biased technological changes, or expansion of the social safety net and the welfare state. All of those could have impacted the earnings process and the reduction

[15] Note that since the level of the series is integrated, we cannot consistently estimate parameters $\{\pi_0, \pi_1\}$ with the OLS regression in Table 1. That is another reason for our preferred choice of shock specification to be the one with the ARIMA-BN process as mentioned in the text. This point is also emphasized in Reis (2009).

of risk in the consumption process, making it a strong assumption that the majority of the factors behind the substantial decrease in $\hat{\sigma}^2_{u,2}$ could be loaded on θ fixed over time.

In order to tackle this challenge, we relax part of our estimation approach and make use of a methodology that allows us to estimate a time-varying, $\hat{\sigma}^2_{u,t}$, and thus a time-varying, $\hat{\theta}_t$. First, we focus only on the post-1947 period, which is the part of the sample that is most relevant to capture movements in the structural parameters of the consumption process. As discussed before, it is well-documented by the literature that the period prior to 1947 was one of incipient stabilization policies and we have already been agnostic in our strategy, within the incipient territory, by using a grid for θ_1 in our previous estimation. Then, inspired by Stock and Watson (2007) analysis of the post-war quarterly inflation process with a parsimonious univariate process, we adapt their methodology to our problem and rewrite our post-war consumption process with a stochastic volatility model. This allows us to work with the desired time-varying variance.

Assume then that the first difference of log-consumption per capita follows a standard stochastic volatility model:

$$\Delta \log(c_t) = \pi_0 + u^c_t, \qquad u^c_t \sim \mathcal{N}(0, e^{h_t}) \tag{31}$$

$$h_t = \mu_h + \phi_h(h_{t-1} - \mu_h) + \varepsilon^h_t, \quad \varepsilon^h_t \sim \mathcal{N}(0, \omega^2_h) \tag{32}$$

where h is the time-varying component of consumption volatility and the process is initialized with $h_1 \sim \mathcal{N}(\mu_h, \omega^2_h/(1-\phi^2_h))$.

We estimate this process with a Bayesian approach using a Markov Chain Monte Carlo (MCMC) method developed by Chan and Grant (2016). We use years 1947-2019 and also multiply the level by 100 for the simulations. To make the computations, we need to start with a choice of suitable priors with a suitable degree of dispersion. Our choices of priors for the parameters are:

$$\pi_0 \sim \mathcal{N}(0, 10) \qquad \mu_h \sim \mathcal{N}(1, 10) \tag{33}$$
$$\phi_h \sim \mathcal{N}(0.9, 0.1^2)\mathbf{1}(|\phi_h| < 1) \qquad \omega_h^2 \sim \mathcal{IG}(10, 0.36)$$

where $\mathcal{IG}(\cdot, \cdot)$ denotes an inverse-gamma distribution.

Table 2 shows the posterior means obtained in the Bayesian estimation of the four parameters used in the process:

Table 2: Posterior means of the stochastic volatility process.

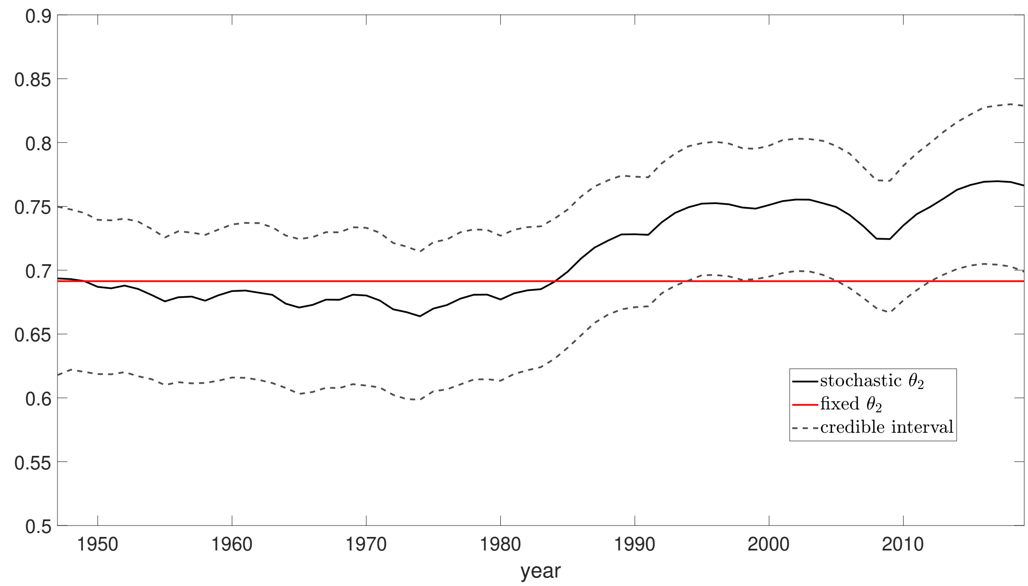
Parameter	Posterior mean	Std. deviation
π_0	1.98	(0.20)
μ_h	0.96	(0.48)
ϕ_h	0.88	(0.08)
ω_h^2	0.04	(0.01)

Notes: The table shows the posterior means and the standard deviations (in parenthesis) for the posterior distributions of the four parameters used in the stochastic volatility process described in equations (31) and (32). Estimates were obtained using the method proposed in Chan and Grant (2016).

Given our estimates, we compute the quantiles 16, 50 and 84 of the posterior distribution of the stochastic volatility, i.e., h^{16}, h^{50}, and h^{84}. We use the median, h^{50}, as the main reference point for the implied time-variant $\hat{\theta}_{2,t}$ and the other quantiles to build the credible interval and generate the dispersion bands. Using the same approach as in the previous exercise, the values reported consider $\theta_1 = 0.2$ and $\hat{\sigma}_{u,1} = 0.0021$. Figure 2 shows our results with the time series for the span:[16]

[16] In Appendix D, we include Figure 4 with the estimated time series for the stochastic standard deviation, $\sqrt{e^{h_t^{50}}}$.

Figure 2: Estimated $\hat{\theta}_{2,t}$ using stochastic volatility.



Notes: The figure shows the estimated time series for $\hat{\theta}_{2,t}$. The solid black line shows the values associated with the median quantile of the estimation, while the dashed lines indicating the bands of the credible interval are associated with quantiles 16 and 84. The solid red line shows the $\hat{\theta}_2$ obtained in the estimation shown in Table 1. These values were obtained from the estimation of the stochastic volatility process for the first difference of log-consumption described in equations (31) and (32). The series spans from 1947 through 2019 and is computed considering $\theta_1 = 0.2$ and $\hat{\sigma}_{u,1} = 0.0021$. In Appendix D, Figure 4 shows the estimated time series for the stochastic standard deviation $\sqrt{e^{h_t^{50}}}$.

We can observe that the time series for the $\hat{\theta}_2$ estimated from the consumption process with stochastic volatility ranges from around 65 percent to 77 percent. These values do not lie far from our original estimated value, 69 percent, with the two-period identification methodology. For a large part of the time series, namely, from the end of WWII until the late 80s, the values for the span remained almost flat, gravitating around the center value of 69 percent. Starting in the early 90s and beyond, the value for time-varying θ starts to climb, potentially settling at a new, higher level, around 75 percent, with a few oscillations around 2010.

These results and the behavior of the series tell us that our initial strategy, along with the estimate we obtained with it, are not far from the ones obtained with a more complex and flexible fitting of the data and can then suffice for being used in our welfare cost calculations. This is not only reassuring but convenient, as our proposed methodology can accommodate only one span at a time. Second, the climb in the value of $\hat{\theta}_2$ starting in the early 1990s until today is consistent with the overall compression of the estimated consumption volatility on this period shown in 2b, which also exhibits an increased persistence, a factor that might be attributed to the "Great Moderation" (Stock and Watson, 2002). This further highlights the existence of ongoing stabilization policies that are reflected in a less volatile consumption process, exactly the effect that the implied θ aims to capture.

7 Welfare Costs of Economic Fluctuations

With the estimated values for $\hat{\theta}_2$ and $\hat{\sigma}_\varepsilon^2$, we can now turn back to the calculation of our decomposition for λ^T, λ^B, and λ^R shown in Theorem 1. We show all our results in Table 3. The numbers are obtained by plugging in the estimates in Table 1 into equations (10) through (20) and shown for all the implied $\hat{\theta}_2$ from our grid for θ_1 and for four different values of the degree of relative risk aversion, γ. For the permanent and ARIMA-BN shocks, the values reported consider $\beta = 0.96$.[17] We also provide a measure that is more naturally comparable to the ones shown in the literature that computes costs with the absence of the span θ, which is represented by λ^{lit} placed in the last column of the table. The derivation of this equivalent cost is straightforward and hence we leave it to Appendix B.4.

[17] In Appendix E.1, we report the results for $\beta \in \{0.95, 0.96, 0.97\}$.

Table 3: Decomposition of the welfare cost of total economic fluctuations.

Transitory shocks													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.37	0.44	0.50	0.56	0.37	0.44	0.50	0.56	0.37	0.44	0.50	0.56	-
$\gamma = 1$	0.32	0.40	0.51	0.66	0.12	0.17	0.25	0.37	0.20	0.23	0.25	0.29	0.13
$\gamma = 2.5$	0.81	1.00	1.27	1.66	0.42	0.58	0.82	1.17	0.39	0.42	0.44	0.48	0.32
$\gamma = 5$	1.63	2.01	2.55	3.35	0.91	1.27	1.78	2.53	0.71	0.74	0.76	0.80	0.64
$\gamma = 7.5$	2.45	3.04	3.86	5.07	1.40	1.96	2.74	3.90	1.03	1.06	1.08	1.12	0.96
Permanent shocks													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.63	0.67	0.70	0.74	0.63	0.67	0.70	0.74	0.63	0.67	0.70	0.74	-
$\gamma = 1$	2.63	3.92	4.99	6.65	1.64	3.02	4.11	5.74	0.97	0.88	0.84	0.86	0.36
$\gamma = 2.5$	3.25	4.89	6.33	8.91	2.15	3.92	5.40	7.97	1.08	0.94	0.88	0.88	0.52
$\gamma = 5$	4.14	6.28	8.34	13.01	2.89	5.21	7.34	11.99	1.21	1.02	0.93	0.90	0.63
$\gamma = 7.5$	5.44	8.39	11.60	15.47	4.00	7.19	10.52	14.39	1.39	1.12	0.98	0.95	0.69
ARIMA-BN													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.61	0.65	0.69	0.73	0.61	0.65	0.69	0.73	0.61	0.65	0.69	0.73	-
$\gamma = 1$	5.15	6.40	8.16	10.79	3.13	4.13	5.58	7.77	1.96	2.18	2.45	2.81	0.75
$\gamma = 2.5$	6.75	8.49	11.06	15.08	5.13	6.73	9.11	12.86	1.54	1.65	1.79	1.96	0.94
$\gamma = 5$	8.16	10.63	14.65	22.16	6.71	9.09	12.96	20.26	1.36	1.41	1.49	1.58	1.03
$\gamma = 7.5$	9.64	13.37	20.92	51.68	8.25	11.88	19.28	49.52	1.29	1.33	1.38	1.44	1.06

Notes: The table displays the computed parameters for the decomposition of the welfare cost of total economic fluctuations. The numbers are obtained using equations (10) through (20) with the estimates shown in Table 1. All of the entries are in percentages of lifetime consumption. We also report an extra welfare cost measure, λ^{lit}, described in Appendix B.4. We report numbers for the relative degree of risk aversion $\gamma \in \{1, 2.5, 5, 7.5\}$, for the implied $\hat{\theta}_2$ along the grid for $\theta_1 \in \{0, 0.1, 0.2, 0.3\}$, and with a calibrated $\beta = 0.96$ for the permanent shocks and ARIMA-BN process.

We focus on the usual level of relative risk aversion used in the literature, $\gamma = 2.5$. For transitory shocks with a span of $\hat{\theta}_2 = 0.50$, the total cost, λ^T, is 1.27 percent of lifetime consumption, being divided into 0.82 stemming from the benefit of current stabilization policies and 0.44 of residual cost. In this case, we recover modest numbers for the costs, though 1.27 percent of lifetime consumption is already at the higher levels estimated in the literature. Nonetheless, the comparison with λ^{lit} implies a small difference of 12 percentage points with λ^R, showing the limitations imposed by the original shock structure.

With permanent shocks we obtain an overall increase of all λ's with a substantially high $\lambda^T = 6.33$, with more than 85 percent of this stemming from λ^B, hence already being stabilized. We end up with 0.88 percent of consumption still left to be smoothed by policies. If we compare this result to the equivalent calculation obtained in the literature, λ^{lit}, we can see that, for our permanent shocks specification, our residual cost is almost 70 percent larger than what a measure with the absence of θ would imply.

Finally, we can focus on our preferred specification of the shocks, the ARIMA-BN process structure, which is the one that better fits the time-series characteristics of the log-consumption data. Results shown in the bottom third of Table 3 point to tge high welfare costs of total economic fluctuations. The total cost, λ^T, is 11 percent of lifetime consumption with $\lambda^B = 9.11$, or 82 percent of it represented as the benefit of ongoing policies. This leaves us with a residual of $\lambda^R = 1.79$ yet to be smoothed, almost double the value of λ^{lit}.

More importantly, beyond finding high levels of costs for λ^T, the approach is able to unveil how much of the total welfare costs are left unaccounted for if one focuses only on the residual measures. Fixing $\gamma = 2.5$, even if we assume a zero effect of stabilization policies in the pre-1947 period, there would still be 5.13 percent of lifetime consumption accruing to the benefit of ongoing policies. Had we assumed a $\theta_1 = 0.3$, the highest value in our grid, we would then jump to almost 13 percent of lifetime consumption smoothed by the stabilization policies that are already set in place. If we return to $\hat{\theta}_2 = 0.69$ and let $\gamma = 5$, we have that the total cost is 14.65 percent of lifetime consumption, out of which 88 percent is already being stabilized.

The results also allow us to explore a simple theoretical aspect that allows us to understand how the concave utility interacts with our proposed decomposition and the parameter θ. If we fix a given level of the measured span of policies, the marginal benefit of smoothing the residual fluctuations in proportion to the total welfare cost, i.e., λ^R/λ^T, is decreasing in the relative degree of risk aversion. Risk-averse consumers tend to value relatively more the benefit generated by the ongoing stabilization policies, going up as much as 92 percent of the total welfare cost with the ARIMA-BN process when γ is at the highest level considered.

8 Robustness - Structural Break

An important potential concern is the presence of structural breaks in a long time series. We have already identified a structural break in the volatility of our consumption series, but it is also important to test whether we find any in the historical path of the consumption data. We apply the methodology developed in Bai and Perron (1998, 2003) to test structural breaks in our sample. For the transitory shock version we test a structural break in the log-consumption and we find a breaks in 1879, 1931, and 1993. We also test a structural break in the first difference of log-consumption, which is the series used in our main analysis that accommodates permanent and persistent shocks. We obtain a scaled F-statistic of 9.31 (with critical value of 8.58), indicating a break in 1934.[18]

We use these breaks to create a new division of the sub-samples that are used in our identification strategy. Since the level and the first difference both indicate a break in the 1930s, this is then the most relevant period for adjustment of the data. The first sub-sample we define considers the years from the beginning of the sample until the years of the structural breaks, and a second sub-sample, as in the main text, considers the years after WWII. To keep the sub-samples the same size for our estimations with all types of shocks, we set the first sub-sample for the years between 1835 and 1930.

The results of the estimated parameters along with the implied $\hat{\theta}_2$ are presented in Table 4. If we compare those results with the results in Table 1 in our main exercise, we note that the estimations imply only a marginal change in the implied parameters with all three shock structures. In Appendix E.2, Tables 7 and 8 present the welfare cost calculations for our λ's using the implied parameters in Table 4. For our preferred parameters, we find a difference of only 1 percentage point for the estimate of $\hat{\theta}_2$ and the computed λ^T, both with a lower value.

[18] In our tests, we allow for at most 5 breaks in the time series. The tests indicate only one break in the first difference of log-consumption (1934) and indicate 3 breaks in the log-consumption (1879, 1931, and 1993). For 1931 the scaled F-statistic is 1221.88 with a critical value of 11.47.

Table 4: Structural break - Estimation

Transitory shocks				Permanent shocks				ARIMA-BN			
Estimated parameters				Estimated parameters				Estimated parameters			
	$\hat{\pi}_0$	$\hat{\pi}_1$	$\hat{\sigma}_u^2$		$\hat{\pi}_0$	$\hat{\pi}_1$	$\hat{\sigma}_u^2$		$\hat{\pi}_0$	$\hat{\phi}_1$	$\hat{\sigma}_u^2$
1835 - 1930	7.726	0.0114	0.0055	1835 - 1930	0.0096	-	0.0019	1835 - 1930	0.0096	-	0.0019
	(.0155)	(.0003)			(.0045)				(.0045)		
1947 - 2019	6.6156	0.0219	0.0025	1947 - 2019	0.0203	-	0.0003	1947 - 2019	0.0189	0.2919	0.0003
	(.0427)	(.0003)			(.0020)				(.0030)	(.1321)	
Implied parameters				Implied parameters				Implied parameters			
θ_1	$\hat{\theta}_2$	$\hat{\alpha}_1$	$\hat{\sigma}_\varepsilon^2$	θ_1	$\hat{\theta}_2$	$\hat{\alpha}_1$	$\hat{\sigma}_\varepsilon^2$	θ_1	$\hat{\theta}_2$	$\hat{\alpha}_1$	$\hat{\sigma}_\varepsilon^2$
0.00	0.3215	0.0222	0.0055	0.00	0.6158	0.0208	0.0019	0.00	0.6011	0.0277	0.0019
0.10	0.3893	0.0222	0.0068	0.10	0.6542	0.0209	0.0024	0.10	0.6410	0.0277	0.0024
0.20	0.4572	0.0222	0.0086	0.20	0.6926	0.0209	0.0030	0.20	0.6809	0.0278	0.0030
0.30	0.5250	0.0222	0.0113	0.30	0.7311	0.0210	0.0040	0.30	0.7208	0.0279	0.0040

Notes: The table displays the estimated parameters obtained by running regression (22) for transitory shocks and regression (25) for permanent and ARIMA-BN shocks. The time series used is our sample of the augmented Barro and Ursúa (2010) data with sub-periods 1835-1930 and 1947-2019. The series is I(1) as identified by the ADF, PP, KPSS, and DF-GLS tests. The first difference of the series is identified, by both the AIC and BIC criteria, as an ARMA (0,0) for the pre-1930 period and an ARMA(1,0) for the subsequent years. The implied parameters are obtained using the formulas described in the text and equation (30).

9 Conclusion

In this paper we revisited the long-standing issue of the welfare costs of business cycles with a focus on unveiling the extent to which ongoing stabilization policies are smoothing observed consumption. We rooted our approach in the novel modelling that all data we gather on consumption are subject to the policy status quo and we provided a decomposition for macroeconomic fluctuations. We recovered the total welfare costs of business cycles by disentangling them into the benefit of current policies and the residual yet to be flattened.

We also conducted an empirical analysis with the goal of identifying our key decomposition parameter, the span of stabilization power, from the historical consumption data. In doing so, we profited from the observation that there is a discontinuous decrease in the series' volatility after WWII, a fact widely documented by a vast literature in macroeconomics. With the proper strategy, we were able to recover estimates from the data and found that the span of stabilization power, in our preferred shock structure and parameter space, is approximately 69 percent and the welfare costs of total economic fluctuations are around 11 percent of permanent consumption, with 9 percent of it already being smoothed by ongoing policies and 1.8 percent left as a residual.

Our paper abstracts from some key aspects that are relevant to our question, such as different types of consumption goods, heterogeneity, and distributional aspects that shed a stronger light on consumption and risk inequality. We also take a simplified view of the role of stabilization policies and technological changes in the post-war US economy. We attempted to tackle part of the latter issue by constructing a time-varying span of stabilization power that yields estimates similar to those in our original analysis. However, we understand that they are all critical considerations that are worth a detailed exploration that could potentially expand our analysis. But for the moment, we leave them for future research.

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Appendix

A Proofs

Below we outline the proofs for Lemmas 1, 2, 3, Propositions 1, 2, 3 and for Theorem 1.

Lemma 1. *Under Assumption 1 and CRRA utility (5),*

$$\sum_{t=0}^{\infty} \beta^t u\left(\bar{C}_t\right) = \begin{cases} \frac{\ln \alpha_0}{1-\beta} + \frac{\beta \ln(1+\alpha_1)}{(1-\beta)^2}, & \text{if } \gamma = 1 \\ \tilde{\alpha}_0 \sum_{t=0}^{\infty} \Gamma^t, & \text{if } \gamma > 1 \end{cases} \tag{34}$$

where $\tilde{\alpha}_0 \equiv (1-\gamma)^{-1}\alpha_0^{1-\gamma}$.

Proof of Lemma 1. Consider a $\gamma = 1$. Then,

$$\sum_{t=0}^{\infty} \beta^t \ln\left(\bar{C}_t\right) = \sum_{t=0}^{\infty} \beta^t \left(\ln \alpha_0 + t \ln\left(1+\alpha_1\right)\right) = \frac{\ln \alpha_0}{1-\beta} + \frac{\beta \ln\left(1+\alpha_1\right)}{(1-\beta)^2}. \tag{35}$$

When $\gamma > 1$,

$$\sum_{t=0}^{\infty} \beta^t (1-\gamma)^{-1} \left(\bar{C}_t\right)^{1-\gamma} = (1-\gamma)^{-1}\alpha_0^{1-\gamma} \sum_{t=0}^{\infty} \left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t = \tilde{\alpha}_0 \sum_{t=0}^{\infty} \Gamma^t. \tag{36}$$

■

Lemma 2. *Consider an arbitrary constant $k > 0$. Under Assumption 1 and CRRA utility (5),*

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty} \beta^t u\left((1+k)\tilde{C}_t\right)\right] = \begin{cases} \frac{\ln(1+k)}{1-\beta} + \frac{\ln \alpha_0}{1-\beta} + \frac{\beta \ln(1+\alpha_1)}{(1-\beta)^2} - \frac{1}{2}\sum_{t=0}^{\infty} \beta^t \sigma_t^2, & \text{if } \gamma = 1 \\ \tilde{\alpha}_0 (1+k)^{1-\gamma} \sum_{t=0}^{\infty} \Gamma^t \exp\left\{-0.5\gamma\left(1-\gamma\right)\sigma_t^2\right\}, & \text{if } \gamma > 1 \end{cases} \tag{37}$$

where $\tilde{\alpha}_0 \equiv (1-\gamma)^{-1}\alpha_0^{1-\gamma}$.

Proof of Lemma 2. For the case where $\gamma = 1$,

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t \ln\left[(1+k)\tilde{C}_t\right]\right] = E_0\left[\sum_{t=0}^{\infty}\beta^t\left(\ln(1+k)+\ln\alpha_0+t\ln(1+\alpha_1)+x_t-\frac{1}{2}\sigma_t^2\right)\right]$$
$$= \sum_{t=0}^{\infty}\beta^t\left(\ln(1+k)+\ln\alpha_0+t\ln(1+\alpha_1)+E_0[x_t]-\frac{1}{2}\sigma_t^2\right)$$
$$= \frac{\ln(1+k)}{1-\beta}+\frac{\ln\alpha_0}{1-\beta}+\frac{\beta\ln(1+\alpha_1)}{(1-\beta)^2}-\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\sigma_t^2,$$

using the fact that $E_0[x_t]=0$.

For the case where $\gamma>1$,

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\frac{\left[(1+k)\tilde{C}_t\right]^{1-\gamma}}{1-\gamma}\right] = (1-\gamma)^{-1}\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left[(1+k)\alpha_0(1+\alpha_1)^t\exp\left\{x_t-0.5\sigma_t^2\right\}\right]^{1-\gamma}\right]$$
$$= \tilde{\alpha}_0(1+k)^{1-\gamma}\sum_{t=0}^{\infty}\left[\beta(1+\alpha_1)^{1-\gamma}\right]^t\times\ldots$$
$$\ldots\exp\left\{-0.5(1-\gamma)\sigma_t^2\right\}E_0\left[\exp\left\{(1-\gamma)x_t\right\}\right].$$

Note that

$$\mathbb{E}_0\left[\exp\left\{(1-\gamma)x_t\right\}\right]=\exp\left\{E_0\left[(1-\gamma)x_t\right]+0.5Var_0\left[(1-\gamma)x_t\right]\right\}=\exp\left\{0.5(1-\gamma)^2\sigma_t^2\right\}.$$

Thus,

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\frac{\left[(1+k)\tilde{C}_t\right]^{1-\gamma}}{1-\gamma}\right] = \tilde{\alpha}_0(1+k)^{1-\gamma}\sum_{t=0}^{\infty}\Gamma^t\exp\left\{-0.5(1-\gamma)\sigma_t^2\right\}\exp\left\{0.5(1-\gamma)^2\sigma_t^2\right\}$$
$$= \tilde{\alpha}_0(1+k)^{1-\gamma}\sum_{t=0}^{\infty}\Gamma^t\exp\left\{-0.5\gamma(1-\gamma)\sigma_t^2\right\}.$$

■

Lemma 3. *Consider an arbitrary constant $\ell>0$. Under Assumption 1 and CRRA utility (5),*

$$\sum_{t=0}^{\infty}\beta^t u\left((1+\ell)C_t(\theta)\right)=$$

$$\begin{cases} \frac{\ln(1+\ell)}{1-\beta} + \frac{\ln \alpha_0}{1-\beta} + \frac{\beta \ln(1+\alpha_1)}{(1-\beta)^2} - \frac{1}{2}(1-\theta)\sum_{t=0}^{\infty}\beta^t\sigma_t^2, & \text{if } \gamma = 1 \\ \tilde{\alpha}_0(1+\ell)^{1-\gamma}\sum_{t=0}^{\infty}\Gamma^t \exp\left\{-0.5(1-\gamma)(1-\theta)(\theta+\gamma-\gamma\theta)\sigma_t^2\right\}, & \text{if } \gamma > 1 \end{cases} \quad (38)$$

where $\tilde{\alpha}_0 \equiv (1-\gamma)^{-1}\alpha_0^{1-\gamma}$.

Proof of Lemma 3. Again, when $\gamma = 1$,

$$\begin{aligned} \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t \ln\left[(1+\ell)C_t(\theta)\right]\right] &= \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t \ln\left[(1+\ell)\alpha_0(1+\alpha_1)^t \exp\left\{(1-\theta)\left[x_t - 0.5\sigma_t^2\right]\right\}\right]\right] \\ &= \sum_{t=0}^{\infty}\beta^t\left(\ln(1+\ell) + \ln\alpha_0 + t\ln(1+\alpha_1) + (1-\theta)\left[E_0[x_t] - 0.5\sigma_t^2\right]\right) \\ &= \frac{\ln(1+\ell)}{1-\beta} + \frac{\ln\alpha_0}{1-\beta} + \frac{\beta\ln(1+\alpha_1)}{(1-\beta)^2} - \frac{1-\theta}{2}\sum_{t=0}^{\infty}\beta^t\sigma_t^2, \end{aligned}$$

given that $E_0[x_t] = 0$. With $\gamma > 1$,

$$\begin{aligned} \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t \frac{[(1+\ell)C_t(\theta)]^{1-\gamma}}{1-\gamma}\right] &= (1-\gamma)^{-1}\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t[(1+\ell)C_t(\theta)]^{1-\gamma}\right] \\ &= (1-\gamma)^{-1}\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left[(1+\ell)\alpha_0(1+\alpha_1)^t + \ldots\right.\right. \\ &\ldots \left.\left.\exp\left\{(1-\theta)\left[x_t - 0.5\sigma_t^2\right]\right\}\right]^{1-\gamma}\right] \\ &= \tilde{\alpha}_0(1+\ell)^{1-\gamma}\sum_{t=0}^{\infty}\left[\beta(1+\alpha_1)^{1-\gamma}\right]^t \times \ldots \\ &\ldots \exp\left\{-0.5(1-\theta)(1-\gamma)\sigma_t^2\right\} E_0\left[\exp\{(1-\theta)(1-\gamma)x_t\}\right]. \end{aligned}$$

Note that

$$\begin{aligned} \mathbb{E}_0\left[\exp\{(1-\theta)(1-\gamma)x_t\}\right] &= \exp\left\{\mathbb{E}_0\left[(1-\theta)(1-\gamma)x_t\right] + 0.5Var_0\left[(1-\theta)(1-\gamma)x_t\right]\right\} \\ &= \exp\left\{0.5(1-\theta)^2(1-\gamma)^2\sigma_t^2\right\}. \end{aligned}$$

And,

$$\begin{aligned}
& \exp\left\{-0.5\left(1-\theta\right)\left(1-\gamma\right)\sigma_t^2\right\}\mathbb{E}_0\left[\exp\left\{\left(1-\theta\right)\left(1-\gamma\right)x_t\right\}\right] \\
= & \exp\left\{-0.5\left(1-\theta\right)\left(1-\gamma\right)\sigma_t^2\right\}\exp\left\{0.5\left(1-\theta\right)^2\left(1-\gamma\right)^2\sigma_t^2\right\} \\
= & \exp\left\{-0.5\left(1-\theta\right)\left(1-\gamma\right)\left(\gamma+\theta-\gamma\theta\right)\sigma_t^2\right\}.
\end{aligned}$$

Thus,

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\frac{\left[(1+\ell)C_t\left(\theta\right)\right]^{1-\gamma}}{1-\gamma}\right]=\tilde{\alpha}_0(1+\ell)^{1-\gamma}\sum_{t=0}^{\infty}\Gamma^t\exp\left\{-0.5\left(1-\theta\right)\left(1-\gamma\right)\left(\gamma+\theta-\gamma\theta\right)\sigma_t^2\right\}$$

■

Proof of Proposition 1. Replace k with λ^B in Lemma 2, use $\ell=0$ in Lemma 3, and then solve equation (3) for λ^B. The assumptions guarantee that $\lambda^B<\infty$. ■

Proof of Proposition 2. We use $\ell=\lambda^R$ in Lemma 3 and the results in Lemma 1 for solving equation (4) for λ^R. The assumptions guarantee that $\lambda^R<\infty$. ■

Proof of Proposition 3. We use $k=\lambda^T$ in Lemma 2 and Lemma 1 in equation (1). Then, we solve it for λ^T. The assumptions guarantee that $\lambda^T<\infty$. ■

Proof of Theorem 1. For $\gamma=1$, we have

$$\begin{aligned}
\left(1+\lambda^B\right)\left(1+\lambda^R\right) &= \exp\left\{\theta\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\sigma_t^2\right\}\exp\left\{\left(1-\theta\right)\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\sigma_t^2\right\} \\
&= \exp\left\{\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\sigma_t^2\right\}=1+\lambda^T
\end{aligned}$$

Now, for $\gamma>1$, we have

$$\left(1+\lambda^{B}\right)^{1-\gamma}\left(1+\lambda^{R}\right)^{1-\gamma} = \frac{\sum_{t=0}^{\infty}\Gamma^{t}e^{-0.5(1-\gamma)(1-\theta)(\theta+\gamma-\gamma\theta)\sigma_t^2}}{\sum_{t=0}^{\infty}\Gamma^{t}e^{-0.5\gamma(1-\gamma)\sigma_t^2}}\frac{\sum_{t=0}^{\infty}\Gamma^{t}}{\sum_{t=0}^{\infty}\Gamma^{t}e^{-0.5(1-\theta)(1-\gamma)(\gamma+\theta-\gamma\theta)\sigma_t^2}}$$

$$\Longleftrightarrow \left(1+\lambda^{B}\right)\left(1+\lambda^{R}\right) = \left[\frac{\sum_{t=0}^{\infty}\Gamma^{t}}{\sum_{t=0}^{\infty}\Gamma^{t}e^{-0.5\gamma(1-\gamma)\sigma_t^2}}\right]^{\frac{1}{1-\gamma}} = 1+\lambda^{T}$$

■

B Calculations for the Applications

B.1 Example 1 (Lucas, 1987):

For $\gamma = 1$:

$$\lambda^{T} = \exp\left\{\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^{t}\sigma_t^2\right\} - 1 = \exp\left\{\frac{1}{2}\sigma_\varepsilon^2\right\} - 1 \tag{39}$$

$$\lambda^{B} = \exp\left\{\theta\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^{t}\sigma_t^2\right\} - 1 = \exp\left\{\theta\frac{1-\beta}{2}\sigma_\varepsilon^2\frac{1}{1-\beta}\right\} - 1 = \exp\left\{\frac{\theta}{2}\sigma_\varepsilon^2\right\} - 1 \tag{40}$$

$$\lambda^{R} = \exp\left\{(1-\theta)\frac{1-\beta}{2}\sigma_\varepsilon^2\frac{1}{1-\beta}\right\} - 1 = \exp\left\{\frac{1-\theta}{2}\sigma_\varepsilon^2\right\} - 1 \tag{41}$$

For $\gamma > 1$:

$$\begin{aligned}
\lambda^{T} &= \left[\frac{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^{t}}{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^{t}\exp\left\{0.5\gamma\left(\gamma-1\right)\sigma_t^2\right\}}\right]^{\frac{1}{1-\gamma}} - 1 \\
&= \left[\frac{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^{t}}{\exp\left\{0.5\gamma\left(\gamma-1\right)\sigma_\varepsilon^2\right\}\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^{t}}\right]^{\frac{1}{1-\gamma}} - 1 \\
&= \left[\frac{1}{\exp\left\{0.5\gamma\left(\gamma-1\right)\sigma_\varepsilon^2\right\}}\right]^{\frac{1}{1-\gamma}} - 1 = \exp\left\{\frac{1}{2}\gamma\sigma_\varepsilon^2\right\} - 1
\end{aligned} \tag{42}$$

$$
\begin{aligned}
\lambda^B &= \left[\frac{\exp\left\{-0.5\left(1-\theta\right)\left(1-\gamma\right)\left[\gamma+\theta-\theta\gamma\right]\sigma^2\right\}\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t}{\exp\left\{0.5\gamma\left(\gamma-1\right)\sigma_\varepsilon^2\right\}\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t}\right]^{\frac{1}{1-\gamma}}-1 \\
&= \left[\frac{\exp\left\{-0.5\left(1-\theta\right)\left(1-\gamma\right)\left[\gamma+\theta-\theta\gamma\right]\sigma_\varepsilon^2\right\}}{\exp\left\{-0.5\gamma\left(1-\gamma\right)\sigma_\varepsilon^2\right\}}\right]^{\frac{1}{1-\gamma}}-1 \\
&= \exp\left\{\frac{1}{2}\left[\gamma-\left(1-\theta\right)\left(\gamma+\theta-\theta\gamma\right)\right]\sigma_\varepsilon^2\right\} \qquad (43)
\end{aligned}
$$

$$
\begin{aligned}
\lambda^R &= \left[\frac{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t}{\exp\left\{-0.5\left(1-\theta\right)\left(1-\gamma\right)\left[\gamma+\theta-\theta\gamma\right]\sigma_\varepsilon^2\right\}\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t}\right]^{\frac{1}{1-\gamma}}-1 \\
&= \left[\frac{1}{\exp\left\{-0.5\left(1-\theta\right)\left(1-\gamma\right)\left[\gamma+\theta-\theta\gamma\right]\sigma_\varepsilon^2\right\}}\right]^{\frac{1}{1-\gamma}}-1 \\
&= \exp\left\{\frac{1}{2}\left(1-\theta\right)\left(\gamma+\theta-\theta\gamma\right)\sigma_\varepsilon^2\right\} \qquad (44)
\end{aligned}
$$

B.2 Example 2 (Obstfeld, 1994)

For $\gamma = 1$:

$$
\begin{aligned}
\lambda^T &= \exp\left\{\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\sigma_t^2\right\}-1 = \exp\left\{\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\left(t\sigma_\varepsilon^2+\sigma_\varepsilon^2\right)\right\}-1 \\
&= \exp\left\{\frac{1-\beta}{2}\left[\frac{\beta}{\left(1-\beta\right)^2}+\frac{1}{1-\beta}\right]\sigma_\varepsilon^2\right\}-1 = \exp\left\{\frac{1}{2}\frac{1}{1-\beta}\sigma_\varepsilon^2\right\}-1 \qquad (45)
\end{aligned}
$$

$$
\begin{aligned}
\lambda^B &= \exp\left\{\theta\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\sigma_t^2\right\}-1 = \exp\left\{\theta\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\left(t\sigma_\varepsilon^2+\sigma_\varepsilon^2\right)\right\}-1 \\
&= \exp\left\{\frac{\theta}{2}\left[\frac{\beta+1-\beta}{1-\beta}\right]\sigma_\varepsilon^2\right\}-1 = \exp\left\{\frac{\theta}{2}\frac{1}{1-\beta}\sigma_\varepsilon^2\right\}-1 \qquad (46)
\end{aligned}
$$

$$\begin{aligned}\lambda^R &= \exp\left\{(1-\theta)\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\sigma_t^2\right\}-1=\exp\left\{(1-\theta)\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\left(t\sigma_\varepsilon^2+\sigma_\varepsilon^2\right)\right\}-1\\ &= \exp\left\{\frac{1-\theta}{2}\left[\frac{\beta+1-\beta}{1-\beta}\right]\sigma_\varepsilon^2\right\}-1=\exp\left\{\frac{1-\theta}{2}\frac{1}{1-\beta}\sigma_\varepsilon^2\right\}-1 \qquad (47)\end{aligned}$$

For $\gamma > 1$:

$$\begin{aligned}\lambda^T &= \left[\frac{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t}{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t\exp\left\{0.5\gamma\left(\gamma-1\right)\left(t\sigma_\varepsilon^2+\sigma_\varepsilon^2\right)\right\}}\right]^{\frac{1}{1-\gamma}}-1\\ &= \left[\frac{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t}{\exp\left\{0.5\gamma\left(\gamma-1\right)\sigma_\varepsilon^2\right\}\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\exp\left\{0.5\gamma\left(\gamma-1\right)\sigma_\varepsilon^2\right\}\right]^t}\right]^{\frac{1}{1-\gamma}}-1\\ &= \left[\frac{1}{1-\beta\left(1+\alpha_1\right)^{1-\gamma}}\frac{1-\beta\left(1+\alpha_1\right)^{1-\gamma}\exp\left\{0.5\gamma\left(\gamma-1\right)\sigma_\varepsilon^2\right\}}{\exp\left\{0.5\gamma\left(\gamma-1\right)\sigma_\varepsilon^2\right\}}\right]^{\frac{1}{1-\gamma}}-1\\ &= \exp\left\{0.5\gamma\sigma_\varepsilon^2\right\}\left[\frac{1-\beta\left(1+\alpha_1\right)^{1-\gamma}\exp\left\{0.5\gamma\left(\gamma-1\right)\sigma_\varepsilon^2\right\}}{1-\beta\left(1+\alpha_1\right)^{1-\gamma}}\right]^{\frac{1}{1-\gamma}}-1 \qquad (48)\end{aligned}$$

$$\begin{aligned}&\lambda^B =\\ &= \left[\frac{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t\exp\left\{-0.5\left(1-\theta\right)\left(1-\gamma\right)\left[\gamma+\theta-\theta\gamma\right]\left(t\sigma_\varepsilon^2+\sigma_\varepsilon^2\right)\right\}}{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t\exp\left\{0.5\gamma\left(\gamma-1\right)\left(t\sigma_\varepsilon^2+\sigma_\varepsilon^2\right)\right\}}\right]^{\frac{1}{1-\gamma}}-1\\ &= \left[\frac{\exp\left\{-0.5\left(1-\theta\right)\left(1-\gamma\right)\left[\gamma+\theta-\theta\gamma\right]\sigma_\varepsilon^2\right\}}{\exp\left\{-0.5\gamma\left(1-\gamma\right)\sigma_\varepsilon^2\right\}}\frac{\frac{1}{1-\beta(1+\alpha_1)^{1-\gamma}\exp\left\{-0.5(1-\theta)(1-\gamma)[\gamma+\theta-\theta\gamma]\sigma_\varepsilon^2\right\}}}{\frac{1}{1-\beta(1+\alpha_1)^{1-\gamma}\exp\left\{0.5\gamma(\gamma-1)\sigma_\varepsilon^2\right\}}}\right]^{\frac{1}{1-\gamma}}-1\\ &= \frac{\exp\left\{0.5\gamma\sigma^2\right\}}{\exp\left\{0.5\left(1-\theta\right)\left[\gamma+\theta-\theta\gamma\right]\sigma^2\right\}}\\ &\quad\times\left[\frac{1-\beta\left(1+\alpha_1\right)^{1-\gamma}\exp\left\{0.5\gamma\left(\gamma-1\right)\sigma_\varepsilon^2\right\}}{1-\beta\left(1+\alpha_1\right)^{1-\gamma}\exp\left\{-0.5\left(1-\theta\right)\left(1-\gamma\right)\left[\gamma+\theta-\theta\gamma\right]\sigma_\varepsilon^2\right\}}\right]^{\frac{1}{1-\gamma}}-1 \qquad (49)\end{aligned}$$

$$
\begin{aligned}
\lambda^{R} &= \left[\frac{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_{1}\right)^{1-\gamma}\right]^{t}}{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_{1}\right)^{1-\gamma}\right]^{t}\exp\left\{-0.5\left(1-\theta\right)\left(1-\gamma\right)\left[\gamma+\theta-\theta\gamma\right]\left(t\sigma_{\varepsilon}^{2}+\sigma_{\varepsilon}^{2}\right)\right\}}\right]^{\frac{1}{1-\gamma}}-1 \\
&= \left[\frac{1}{\exp\left\{-0.5\left(1-\theta\right)\left(1-\gamma\right)\left[\gamma+\theta-\theta\gamma\right]\sigma_{\varepsilon}^{2}\right\}}\right]^{\frac{1}{1-\gamma}} \\
&\quad\times\left[\frac{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_{1}\right)^{1-\gamma}\right]^{t}}{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_{1}\right)^{1-\gamma}\exp\left\{-0.5\left(1-\theta\right)\left(1-\gamma\right)\left[\gamma+\theta-\theta\gamma\right]\sigma_{\varepsilon}^{2}\right\}\right]^{t}}\right]^{\frac{1}{1-\gamma}}-1 \\
&= \frac{1}{\exp\left\{-0.5\left(1-\theta\right)\left[\gamma+\theta-\theta\gamma\right]\sigma_{\varepsilon}^{2}\right\}} \\
&\quad\times\left[\frac{1-\beta\left(1+\alpha_{1}\right)^{1-\gamma}\exp\left\{-0.5\left(1-\theta\right)\left(1-\gamma\right)\left[\gamma+\theta-\theta\gamma\right]\sigma_{\varepsilon}^{2}\right\}}{1-\beta\left(1+\alpha_{1}\right)^{1-\gamma}}\right]^{\frac{1}{1-\gamma}}-1 \qquad (50)
\end{aligned}
$$

B.3 Example 3 - ARIMA-BN Process (Reis, 2009):

From the Beveridge-Nelson decomposition,

$$
\begin{aligned}
x_{t}^{BN} &= \psi\left(1\right)\sum_{j=0}^{t}\varepsilon_{j}+\sum_{j=0}^{t}\varphi_{j}\varepsilon_{t-j} \\
&= \left[\psi\left(1\right)+\varphi_{t}\right]\varepsilon_{0}+\left[\psi\left(1\right)+\varphi_{t-1}\right]\varepsilon_{1}+\cdots+\left[\psi\left(1\right)+\varphi_{1}\right]\varepsilon_{t-1}+\left[\psi\left(1\right)+\varphi_{0}\right]\varepsilon_{t} \\
&= \sum_{j=0}^{t}\left[\psi\left(1\right)+\varphi_{t-j}\right]\varepsilon_{j} \qquad (51)
\end{aligned}
$$

Since ε_0 is revealed at the end of $t=0$, $\mathbb{E}_0\left[x_t^{BN}\right]=0$. Hence,

$$
\begin{aligned}
\sigma_{x_{t}^{BN}}^{2} &\equiv \mathbb{E}\left[\left(x_{t}^{BN}-\mathbb{E}_{0}\left[x_{t}^{BN}\right]\right)^{2}\right]=\mathbb{E}\left[\left(x_{t}^{BN}\right)^{2}\right]=\mathbb{E}\left[\sum_{j=0}^{t}\left[\psi\left(1\right)+\varphi_{t-j}\right]^{2}\varepsilon_{j}^{2}\right] \\
&= \sum_{j=0}^{t}\left[\psi\left(1\right)^{2}+2\psi\left(1\right)\varphi_{t-j}+\varphi_{t-j}^{2}\right]\sigma_{\varepsilon}^{2} \\
&= \left(t+1\right)\psi\left(1\right)^{2}\sigma_{\varepsilon}^{2}+2\psi\left(1\right)\sum_{j=0}^{t}\varphi_{t-j}\sigma_{\varepsilon}^{2}+\sum_{j=0}^{t}\varphi_{t-j}^{2}\sigma_{\varepsilon}^{2} \qquad (52)
\end{aligned}
$$

which can be rewritten into (17).

Hence, for $\gamma = 1$,

$$\begin{aligned}\lambda^T &= \exp\left\{\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\sigma_t^2\right\} - 1 = \exp\left\{\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\left(\rho_0+\rho_1 t\right)\right\} - 1 \\ &= \exp\left\{\frac{1}{2}\left(\rho_0 + \frac{\beta}{1-\beta}\rho_1\right)\right\} - 1 \end{aligned} \tag{53}$$

$$\lambda^B = \exp\left\{\theta\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\left(\rho_0+\rho_1 t\right)\right\} - 1 = \exp\left\{\frac{\theta}{2}\left(\rho_0 + \frac{\beta}{1-\beta}\rho_1\right)\right\} - 1 \tag{54}$$

$$\lambda^R = \exp\left\{\left(1-\theta\right)\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\left(\rho_0+\rho_1 t\right)\right\} - 1 = \exp\left\{\frac{1-\theta}{2}\left(\rho_0 + \frac{\beta}{1-\beta}\rho_1\right)\right\} - 1 \tag{55}$$

For $\gamma > 1$,

$$\begin{aligned}\lambda^T &= \left[\frac{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t}{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t\exp\left\{-0.5\gamma\left(1-\gamma\right)\left(\rho_0+\rho_1 t\right)\right\}}\right]^{\frac{1}{1-\gamma}} - 1 \\ &= \left[\frac{1}{\exp\left\{-0.5\gamma\left(1-\gamma\right)\rho_0\right\}}\right]^{\frac{1}{1-\gamma}} \\ &\quad \times \left[\frac{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t}{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\exp\left\{-0.5\gamma\left(1-\gamma\right)\rho_1\right\}\right]^t}\right]^{\frac{1}{1-\gamma}} \\ &= \exp\left\{0.5\gamma\rho_0\right\}\left[\frac{1-\beta\left(1+\alpha_1\right)^{1-\gamma}\exp\left\{-0.5\gamma\left(1-\gamma\right)\rho_1\right\}}{1-\beta\left(1+\alpha_1\right)^{1-\gamma}}\right]^{\frac{1}{1-\gamma}} \end{aligned} \tag{56}$$

$$
\begin{aligned}
\lambda^B &= \left[\frac{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t \exp\left\{-0.5\left(1-\gamma\right)\left(1-\theta\right)\left(\theta+\gamma-\gamma\theta\right)\left(\rho_0+\rho_1 t\right)\right\}}{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t \exp\left\{-0.5\gamma\left(1-\gamma\right)\left(\rho_0+\rho_1 t\right)\right\}}\right]^{\frac{1}{1-\gamma}} - 1 \\
&= \left[\frac{\exp\left\{-0.5\left(1-\gamma\right)\left(1-\theta\right)\left(\theta+\gamma-\gamma\theta\right)\rho_0\right\}}{\exp\left\{-0.5\gamma\left(1-\gamma\right)\rho_0\right\}}\right]^{\frac{1}{1-\gamma}} \\
&\quad \times \left[\frac{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\exp\left\{-0.5\left(1-\gamma\right)\left(1-\theta\right)\left(\theta+\gamma-\gamma\theta\right)\rho_1\right\}\right]^t}{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\exp\left\{-0.5\gamma\left(1-\gamma\right)\rho_1\right\}\right]^t}\right]^{\frac{1}{1-\gamma}} - 1 \\
&= \frac{\exp\left\{0.5\gamma\rho_0\right\}}{\exp\left\{0.5\left(1-\theta\right)\left(\theta+\gamma-\gamma\theta\right)\rho_0\right\}} \\
&\quad \times \left[\frac{1-\beta\left(1+\alpha_1\right)^{1-\gamma}\exp\left\{-0.5\gamma\left(1-\gamma\right)\rho_1\right\}}{1-\beta\left(1+\alpha_1\right)^{1-\gamma}\exp\left\{-0.5\left(1-\gamma\right)\left(1-\theta\right)\left(\theta+\gamma-\gamma\theta\right)\rho_1\right\}}\right]^{\frac{1}{1-\gamma}} - 1 \qquad (57)
\end{aligned}
$$

$$
\begin{aligned}
\lambda^R &= \left[\frac{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t}{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t \exp\left\{-0.5\left(1-\gamma\right)\left(1-\theta\right)\left(\theta+\gamma-\gamma\theta\right)\left(\rho_0+\rho_1 t\right)\right\}}\right]^{\frac{1}{1-\gamma}} - 1 \\
&= \left[\frac{1}{\exp\left\{-0.5\left(1-\gamma\right)\left(1-\theta\right)\left(\theta+\gamma-\gamma\theta\right)\rho_0\right\}}\right]^{\frac{1}{1-\gamma}} \\
&\quad \times \left[\frac{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t}{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\exp\left\{-0.5\left(1-\gamma\right)\left(1-\theta\right)\left(\theta+\gamma-\gamma\theta\right)\rho_1\right\}\right]^t}\right]^{\frac{1}{1-\gamma}} - 1 \\
&= \exp\left\{0.5\left(1-\theta\right)\left(\theta+\gamma-\gamma\theta\right)\rho_0\right\} \\
&\quad \times \left[\frac{1-\beta\left(1+\alpha_1\right)^{1-\gamma}\exp\left\{-0.5\left(1-\gamma\right)\left(1-\theta\right)\left(\theta+\gamma-\gamma\theta\right)\rho_1\right\}}{1-\beta\left(1+\alpha_1\right)^{1-\gamma}}\right]^{\frac{1}{1-\gamma}} - 1
\end{aligned}
$$

B.4 The Literature-based Cost λ^{lit}

Here we characterize in our three applications the welfare cost of business cycles in the absence of observed consumption as proposed in our decomposition. We simply substitute σ_ε^2 by σ_u^2 in our previous calculations and use the formula for λ^T for each type of shock. Recall that, in our methodology, $\sigma_u^2 = (1-\theta_2)^2\sigma_\varepsilon^2$.

Example 1 (Lucas, 1987) :

$$\lambda^{lit} = \begin{cases} \exp\left(\frac{\sigma_u^2}{2}\right) - 1, \text{ if } \gamma = 1 \\ \exp\left(\frac{\gamma\sigma_u^2}{2}\right) - 1, \text{ if } \gamma > 1 \end{cases} \tag{58}$$

Example 2 (Obstfeld, 1994) :

$$\lambda^{lit} = \begin{cases} \exp\left(\frac{\sigma_u^2}{2(1-\beta)}\right) - 1, \text{ if } \gamma = 1 \\ \exp\left\{0.5\gamma\sigma_u^2\right\}\left[\frac{1-\Gamma\exp -0.5\gamma(1-\gamma)\sigma_u^2}{1-\Gamma}\right]^{\frac{1}{1-\gamma}} - 1, \text{ if } \gamma > 1 \end{cases} \tag{59}$$

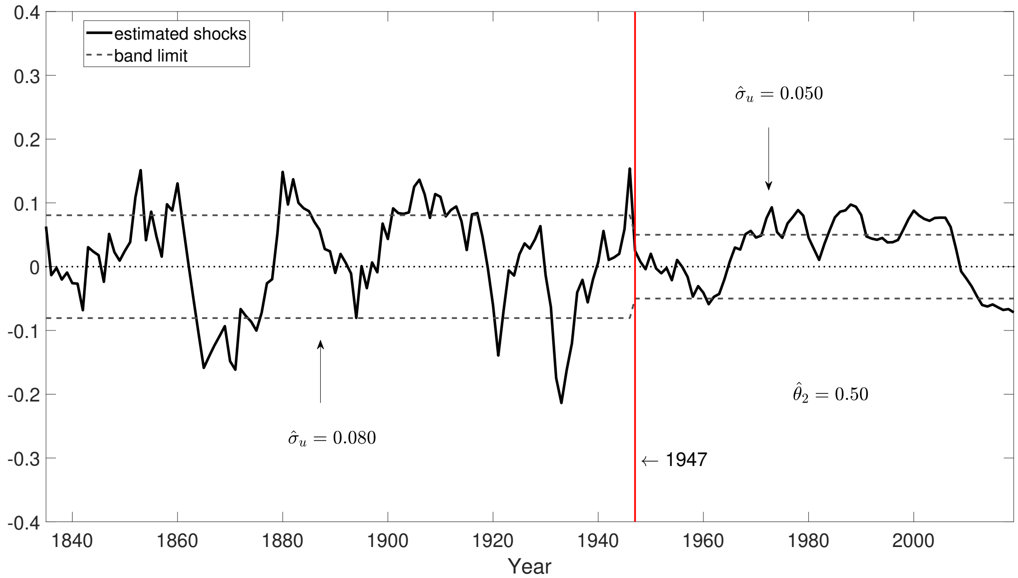
Example 3 - ARIMA-BN Process (Reis, 2009):

In this case, the substitution is in equation (17).

$$\lambda^{lit} = \begin{cases} \exp\left\{\frac{1}{2}\left(\rho_0 + \frac{\beta}{1-\beta}\rho_1\right)\right\} - 1, & \text{if } \gamma = 1 \\ \exp\left\{0.5\gamma\rho_0\right\}\left[\frac{1-\Gamma\exp\left\{-0.5\gamma\left(1-\gamma\right)\rho_1\right\}}{1-\Gamma}\right]\frac{1}{1-\gamma}, & \text{if } \gamma > 1 \end{cases} \tag{60}$$

C Identification in Transitory Shocks

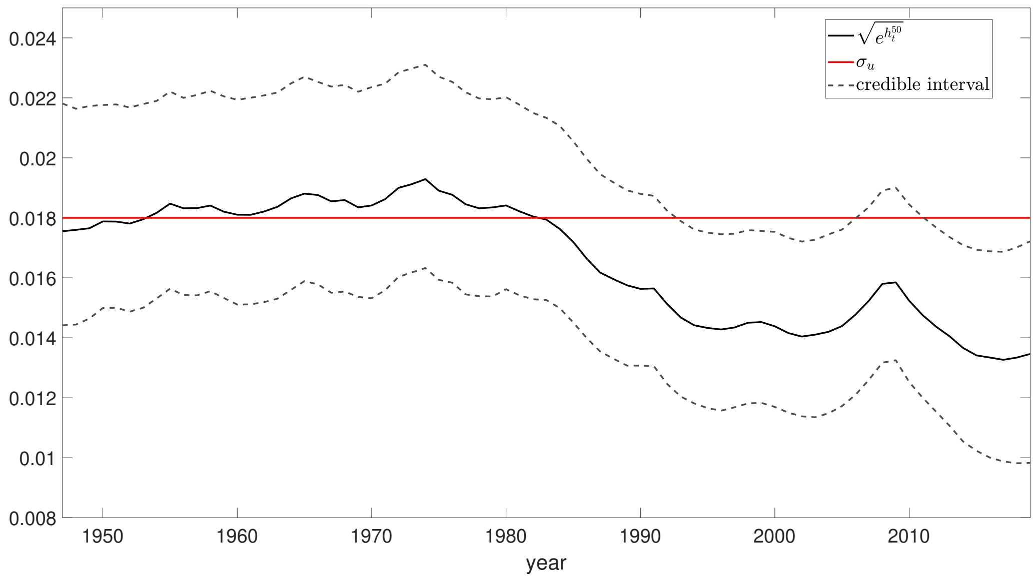
Figure 3: Estimated residuals of transitory shocks between 1835 and 2019.



Notes: The figure shows the time series for per capita consumption for the US between 1835 and 2019 with our augmented sample of the Barro and Ursúa (2010) data. The vertical line marks the year 1947, at the end of WWII. We report the standard errors for the two sub-periods generated by this line along with the average and band limits equivalent to $2\sigma_u$.

D Extra Figure for the Time-Varying θ

Figure 4: Estimated $\sqrt{e^{h_t^{50}}}$ for the stochastic volatility model.



Notes: The figure shows the estimated time series for $\sqrt{e^{h_t^{50}}}$. The solid black line shows the values associated with the median quantile of the estimation, while the dashed lines indicating the bands of the credible interval are associated with quantiles 16 and 84. The solid red line shows the $\hat{\sigma}_u$ obtained in the estimation shown in Table 1. These values were obtained from the estimation of the stochastic volatility process for the first difference of log-consumption described in equations (31) and (32). The series spans from 1947 through 2019 and is computed considering $\theta_1 = 0.2$ and $\hat{\sigma}_{u,1} = 0.0021$.

E Estimates of Welfare Costs for Different β's

E.1 Full Sample

In Tables 5 and 6 we present our estimations of the welfare cost using the full sample as in the main text. For the case of permanent and ARIMA-BN shocks, we compute the λ's for different values of β.

Table 5: Welfare cost - Full sample

Transitory shocks

	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.37	0.44	0.50	0.56	0.37	0.44	0.50	0.56	0.37	0.44	0.50	0.56	-
$\gamma = 1$	0.32	0.40	0.51	0.66	0.12	0.17	0.25	0.37	0.20	0.23	0.25	0.29	0.13
$\gamma = 2.5$	0.81	1.00	1.27	1.66	0.42	0.58	0.82	1.17	0.39	0.42	0.44	0.48	0.32
$\gamma = 5$	1.63	2.01	2.55	3.35	0.91	1.27	1.78	2.53	0.71	0.74	0.76	0.80	0.64
$\gamma = 7.5$	2.45	3.04	3.86	5.07	1.40	1.96	2.74	3.90	1.03	1.06	1.08	1.12	0.96

Permanent shocks

$\beta = 0.95$

	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.63	0.67	0.70	0.74	0.63	0.67	0.70	0.74	0.63	0.67	0.70	0.74	-
$\gamma = 1$	2.10	2.60	3.30	4.33	1.31	1.72	2.31	3.18	0.77	0.86	0.97	1.11	0.29
$\gamma = 2.5$	3.42	4.26	5.46	7.27	2.63	3.41	4.53	6.23	0.77	0.82	0.89	0.98	0.46
$\gamma = 5$	4.58	5.79	7.60	10.51	3.77	4.94	6.69	9.51	0.78	0.81	0.86	0.91	0.58
$\gamma = 7.5$	5.44	7.03	9.56	14.12	4.61	6.16	8.63	13.11	0.80	0.82	0.86	0.90	0.65

$\beta = 0.96$

	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.63	0.67	0.70	0.74	0.63	0.67	0.70	0.74	0.63	0.67	0.70	0.74	-
$\gamma = 1$	2.63	3.92	4.99	6.65	1.64	3.02	4.11	5.74	0.97	0.88	0.84	0.86	0.36
$\gamma = 2.5$	3.25	4.89	6.33	8.91	2.15	3.92	5.40	7.97	1.08	0.94	0.88	0.88	0.52
$\gamma = 5$	4.14	6.28	8.34	13.01	2.89	5.21	7.34	11.99	1.21	1.02	0.93	0.90	0.63
$\gamma = 7.5$	5.44	8.39	11.60	15.47	4.00	7.19	10.52	14.39	1.39	1.12	0.98	0.95	0.69

$\beta = 0.97$

	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.63	0.67	0.70	0.74	0.63	0.67	0.70	0.74	0.63	0.67	0.70	0.74	-
$\gamma = 1$	3.52	4.36	5.55	7.31	2.20	2.88	3.87	5.36	1.29	1.44	1.62	1.85	0.48
$\gamma = 2.5$	4.60	5.74	7.40	9.92	3.54	4.60	6.14	8.51	1.02	1.09	1.18	1.30	0.61
$\gamma = 5$	5.48	6.97	9.23	12.96	4.52	5.96	8.14	11.76	0.92	0.96	1.01	1.07	0.69
$\gamma = 7.5$	6.23	8.13	11.21	17.13	5.29	7.14	10.16	15.97	0.89	0.92	0.96	1.00	0.73

Notes: The table displays the computed parameters for the decomposition of the welfare cost of total economic fluctuations expanding the one in the main text for different β's. The numbers are obtained using equations (10) through (15) with the estimates shown in Table 1. All of the entries are in percentages of lifetime consumption. We also report an extra welfare cost measure, λ^{lit}, described in Appendix B.4. We report numbers for the relative degree of risk aversion $\gamma \in \{1, 2.5, 5, 7.5\}$, for the implied $\hat{\theta}_2$ along the grid for $\theta_1 \in \{0, 0.1, 0.2, 0.3\}$, and with $\beta \in \{0.95, 0.96, 0.97\}$ for the permanent shocks.

Table 6: Welfare cost - Full sample - ARIMA-BN

	$\beta = 0.95$												
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.61	0.65	0.69	0.73	0.61	0.65	0.69	0.73	0.61	0.65	0.69	0.73	-
$\gamma = 1$	4.07	5.05	6.43	8.48	2.48	3.27	4.40	6.12	1.55	1.72	1.94	2.22	0.60
$\gamma = 2.5$	5.91	7.42	9.63	13.05	4.49	5.88	7.92	11.13	1.36	1.46	1.58	1.73	0.83
$\gamma = 5$	7.50	9.72	13.29	19.77	6.16	8.30	11.74	18.03	1.26	1.32	1.38	1.47	0.96
$\gamma = 7.5$	8.99	12.35	18.89	40.20	7.68	10.95	17.35	38.31	1.22	1.26	1.31	1.37	1.01
	$\beta = 0.96$												
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.61	0.65	0.69	0.73	0.61	0.65	0.69	0.73	0.61	0.65	0.69	0.73	-
$\gamma = 1$	5.15	6.40	8.16	10.79	3.13	4.13	5.58	7.77	1.96	2.18	2.45	2.81	0.75
$\gamma = 2.5$	6.75	8.49	11.06	15.08	5.13	6.73	9.11	12.86	1.54	1.65	1.79	1.96	0.94
$\gamma = 5$	8.16	10.63	14.65	22.16	6.71	9.09	12.96	20.26	1.36	1.41	1.49	1.58	1.03
$\gamma = 7.5$	9.64	13.37	20.92	51.68	8.25	11.88	19.28	49.52	1.29	1.33	1.38	1.44	1.06
	$\beta = 0.97$												
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.61	0.65	0.69	0.73	0.61	0.65	0.69	0.73	0.61	0.65	0.69	0.73	-
$\gamma = 1$	6.98	8.69	11.12	14.76	4.23	5.59	7.56	10.57	2.64	2.93	3.31	3.79	1.01
$\gamma = 2.5$	7.84	9.91	12.95	17.80	5.96	7.86	10.68	15.19	1.78	1.90	2.06	2.26	1.08
$\gamma = 5$	8.93	11.71	16.29	25.19	7.36	10.03	14.46	23.09	1.46	1.53	1.61	1.71	1.11
$\gamma = 7.5$	10.38	14.56	23.47	85.20	8.90	12.97	21.70	82.42	1.36	1.40	1.45	1.52	1.12

Notes: The table displays the computed parameters for the decomposition of the welfare cost of total economic fluctuations expanding the one in the main text for different β's. The numbers are obtained using equations (18) through (20) with the estimates shown in Table 1. All of the entries are in percentages of lifetime consumption. We also report an extra welfare cost measure, λ^{lit}, described in Appendix B.4. We report numbers for the relative degree of risk aversion $\gamma \in \{1, 2.5, 5, 7.5\}$, for the implied $\hat{\theta}_2$ along the grid for $\theta_1 \in \{0, 0.1, 0.2, 0.3\}$, and with $\beta \in \{0.95, 0.96, 0.97\}$ for the ARIMA-BN process.

E.2 Structural Break

In Tables 7 and 8 we present our estimations of the welfare cost using the sample adjusted for structural breaks as described in the main text. For the case of permanent and ARIMA-BN shocks, we compute the λ's for different values of β.

Table 7: Structural break - Welfare cost

Transitory shocks													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.32	0.39	0.46	0.53	0.32	0.39	0.46	0.53	0.32	0.39	0.46	0.53	-
$\gamma = 1$	0.28	0.34	0.43	0.56	0.09	0.13	0.20	0.30	0.19	0.21	0.23	0.27	0.13
$\gamma = 2.5$	0.69	0.85	1.08	1.42	0.31	0.45	0.65	0.95	0.38	0.40	0.43	0.46	0.32
$\gamma = 5$	1.39	1.72	2.18	2.85	0.69	0.99	1.42	2.06	0.70	0.72	0.74	0.78	0.64
$\gamma = 7.5$	2.09	2.58	3.28	4.31	1.06	1.53	2.20	3.18	1.02	1.04	1.06	1.10	0.96
Permanent shocks													
$\beta = 0.95$													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.62	0.65	0.69	0.73	0.62	0.65	0.69	0.73	0.62	0.65	0.69	0.73	-
$\gamma = 1$	1.96	2.43	3.08	4.04	1.20	1.58	2.12	2.94	0.75	0.83	0.94	1.07	0.29
$\gamma = 2.5$	3.19	3.97	5.09	6.76	2.42	3.14	4.18	5.75	0.75	0.80	0.87	0.96	0.46
$\gamma = 5$	4.25	5.37	7.03	9.67	3.46	4.53	6.13	8.69	0.77	0.80	0.84	0.90	0.58
$\gamma = 7.5$	4.91	6.32	8.51	12.36	4.09	5.46	7.61	11.38	0.79	0.81	0.84	0.88	0.65
$\beta = 0.96$													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.62	0.65	0.69	0.73	0.62	0.65	0.69	0.73	0.62	0.65	0.69	0.73	-
$\gamma = 1$	2.46	3.66	4.63	6.09	1.51	2.78	3.77	5.20	0.94	0.86	0.83	0.85	0.36
$\gamma = 2.5$	3.04	4.56	5.86	8.09	1.98	3.61	4.95	7.16	1.04	0.92	0.87	0.87	0.52
$\gamma = 5$	3.86	5.85	7.70	11.59	2.66	4.81	6.73	10.60	1.17	0.99	0.91	0.90	0.63
$\gamma = 7.5$	4.98	7.63	10.40	13.46	3.60	6.48	9.34	12.41	1.33	1.08	0.96	0.93	0.69
$\beta = 0.97$													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.62	0.65	0.69	0.73	0.62	0.65	0.69	0.73	0.62	0.65	0.69	0.73	-
$\gamma = 1$	3.29	4.08	5.19	6.83	2.01	2.65	3.56	4.95	1.25	1.39	1.57	1.79	0.48
$\gamma = 2.5$	4.29	5.35	6.88	9.21	3.25	4.24	5.66	7.84	1.00	1.07	1.16	1.27	0.61
$\gamma = 5$	5.08	6.45	8.51	11.87	4.14	5.46	7.45	10.71	0.90	0.94	0.99	1.05	0.69
$\gamma = 7.5$	5.61	7.27	9.92	14.79	4.69	6.30	8.90	13.67	0.88	0.91	0.94	0.99	0.73

Notes: The table displays the computed parameters for the decomposition of the welfare cost of total economic fluctuations using a robustness sample that avoids the structural break in 1931. The numbers are obtained using equations (10) through (15) with the estimates shown in Table 4. All of the entries are in percentages of lifetime consumption. We also report an extra welfare cost measure, λ^{lit}, described in Appendix B.4. We report numbers for the relative degree of risk aversion $\gamma \in \{1, 2.5, 5, 7.5\}$, for the implied $\hat{\theta}_2$ along the grid for $\theta_1 \in \{0, 0.1, 0.2, 0.3\}$, and with $\beta \in \{0.95, 0.96, 0.97\}$ for the permanent shocks.

Table 8: Structural break - Welfare cost - ARIMA-BN

$\beta = 0.95$													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.60	0.64	0.68	0.72	0.60	0.64	0.68	0.72	0.60	0.64	0.68	0.72	-
$\gamma = 1$	3.80	4.71	6.00	7.91	2.27	3.00	4.05	5.64	1.50	1.67	1.88	2.15	0.60
$\gamma = 2.5$	5.50	6.90	8.94	12.08	4.12	5.40	7.28	10.21	1.33	1.43	1.54	1.69	0.83
$\gamma = 5$	6.92	8.94	12.13	17.79	5.60	7.54	10.62	16.11	1.25	1.30	1.36	1.45	0.96
$\gamma = 7.5$	8.19	11.12	16.56	31.28	6.90	9.75	15.07	29.52	1.21	1.25	1.30	1.35	1.01
$\beta = 0.96$													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.60	0.64	0.68	0.72	0.60	0.64	0.68	0.72	0.60	0.64	0.68	0.72	-
$\gamma = 1$	4.81	5.97	7.62	10.06	2.86	3.79	5.13	7.16	1.89	2.10	2.37	2.71	0.75
$\gamma = 2.5$	6.28	7.89	10.25	13.93	4.70	6.18	8.36	11.79	1.51	1.62	1.75	1.92	0.94
$\gamma = 5$	7.52	9.76	13.33	19.83	6.10	8.25	11.69	18.00	1.34	1.40	1.47	1.56	1.03
$\gamma = 7.5$	8.76	11.99	18.17	37.05	7.39	10.53	16.58	35.12	1.28	1.31	1.36	1.43	1.06
$\beta = 0.97$													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.60	0.64	0.68	0.72	0.60	0.64	0.68	0.72	0.60	0.64	0.68	0.72	-
$\gamma = 1$	6.51	8.10	10.36	13.75	3.87	5.12	6.95	9.73	2.55	2.84	3.20	3.66	1.01
$\gamma = 2.5$	7.29	9.19	11.99	16.41	5.46	7.20	9.78	13.90	1.74	1.86	2.01	2.21	1.08
$\gamma = 5$	8.22	10.73	14.78	22.39	6.68	9.08	12.99	20.37	1.44	1.51	1.58	1.68	1.11
$\gamma = 7.5$	9.41	13.00	20.15	46.47	7.96	11.45	18.44	44.30	1.35	1.39	1.44	1.50	1.12

Notes: The table displays the computed parameters for the decomposition of the welfare cost of total economic fluctuations using a robustness sample that avoids the structural break in 1931. The numbers are obtained using equations (18) through (20) with the estimates shown in Table 4. All of the entries are in percentages of lifetime consumption. We also report an extra welfare cost measure, λ^{lit}, described in Appendix B.4. We report numbers for the relative degree of risk aversion $\gamma \in \{1, 2.5, 5, 7.5\}$, for the implied $\hat{\theta}_2$ along the grid for $\theta_1 \in \{0, 0.1, 0.2, 0.3\}$, and with $\beta \in \{0.95, 0.96, 0.97\}$ for the ARIMA-BN process.

F Extra Robustness Exercises

F.1 Removing the Interwar Period

As the previous exercise used the disjoint periods (1835-1930 and 1947-2019), we run an additional experiment where we use the 1931 break in the time series as a reference point to design two new intervals. In the previous exercise, we have removed 15 periods - years 1931 to 1945 - from the full sample. Those periods were exclusively defined after the break. For this case, we remove a similar interval for the period before the 1931 break. We construct two sub-samples by excluding the interwar period from our data, which results in a first period with years 1835 to 1913 and a second period from 1947 to 2019, the last one as in our main analysis.[19]

Besides the structural break in the consumption series during the interwar period, many other relevant macroeconomic events happened during this window of time. For example, we have the 1929 crisis and the Great Depression that followed. In general, this period was marked by highly unstable macroeconomic outcomes, and hence, it is worth subtracting it from the sample to better measure pre-war volatility. Once again, the results are similar to those of our original analysis. Table 9 presents the estimated and implied parameters and Tables 10 and 11 present the computed λ's using the estimations in Table 9. Similarly to the robustness check with structural breaks, there is no substantial change in the results, with our preferred total cost being roughly 1 percentage point smaller than the one shown in our main exercise.

[19] We also run an experiment by removing exactly 15 periods before and after the break, that is, using sub-samples from 1835-1915 and 1947-2019. As expected, the results are so similar to the results in this subsection that we only report the exercise where we remove the interwar period.

Table 9: Removing the interwar period - Estimation

Transitory shocks				Permanent shocks				ARIMA-BN			
Estimated parameters				Estimated parameters				Estimated parameters			
	$\hat{\pi}_0$	$\hat{\pi}_1$	$\hat{\sigma}_u^2$		$\hat{\pi}_0$	$\hat{\pi}_1$	$\hat{\sigma}_u^2$		$\hat{\pi}_0$	$\hat{\phi}_1$	$\hat{\sigma}_u^2$
1835 - 1913	7.7099	0.0119	0.0057	1835 - 1913	0.0107	-	0.0019	1835 - 1913	0.0107	-	0.0019
	(.0175)	(.0004)			(.0049)				(.0049)		
1947 - 2019	6.6156	0.0219	0.0025	1947 - 2019	0.0203	-	0.0003	1947 - 2019	0.0189	0.2919	0.0003
	(.0427)	(.0003)			(.0020)				(.0030)	(.1321)	
Implied parameters				Implied parameters				Implied parameters			
θ_1	$\hat{\theta}_2$	$\hat{\alpha}_1$	$\hat{\sigma}_\varepsilon^2$	θ_1	$\hat{\theta}_2$	$\hat{\alpha}_1$	$\hat{\sigma}_\varepsilon^2$	θ_1	$\hat{\theta}_2$	$\hat{\alpha}_1$	$\hat{\sigma}_\varepsilon^2$
0.00	0.3309	0.0222	0.0057	0.00	0.6120	0.0208	0.0019	0.00	0.5972	0.0277	0.0019
0.10	0.3978	0.0222	0.0070	0.10	0.6508	0.0209	0.0023	0.10	0.6375	0.0277	0.0023
0.20	0.4647	0.0222	0.0089	0.20	0.6896	0.0209	0.0030	0.20	0.6777	0.0278	0.0030
0.30	0.5316	0.0222	0.0116	0.30	0.7284	0.0210	0.0039	0.30	0.7180	0.0279	0.0039

Notes: The table displays the estimated parameters obtained by running regression (22) for transitory shocks and regression (25) for permanent and ARIMA-BN shocks. The time series used is our sample of the augmented Barro and Ursúa (2010) data excluding the interwar period. The series is I(1) as identified by the ADF, PP, KPSS, and DF-GLS tests. The first-difference of the series is identified, by both the AIC and BIC criteria, as an ARMA (0,0) for the pre-1947 period and an ARMA(1,0) for the subsequent years. The implied parameters are obtained using the formulas described in the text and equation (30).

Table 10: Removing the interwar period - Welfare cost

Transitory shocks													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.33	0.40	0.46	0.53	0.33	0.40	0.46	0.53	0.33	0.40	0.46	0.53	-
$\gamma = 1$	0.28	0.35	0.44	0.58	0.09	0.14	0.21	0.31	0.19	0.21	0.24	0.27	0.13
$\gamma = 2.5$	0.71	0.88	1.11	1.46	0.33	0.47	0.68	0.99	0.38	0.40	0.43	0.46	0.32
$\gamma = 5$	1.43	1.76	2.24	2.93	0.72	1.04	1.48	2.14	0.70	0.72	0.75	0.78	0.64
$\gamma = 7.5$	2.15	2.66	3.38	4.43	1.12	1.60	2.29	3.30	1.02	1.04	1.07	1.10	0.96
Permanent shocks													
$\beta = 0.95$													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.61	0.65	0.69	0.73	0.61	0.65	0.69	0.73	0.61	0.65	0.69	0.73	-
$\gamma = 1$	1.92	2.38	3.02	3.96	1.17	1.54	2.07	2.87	0.74	0.82	0.93	1.06	0.29
$\gamma = 2.5$	3.13	3.89	4.98	6.62	2.36	3.07	4.08	5.62	0.75	0.80	0.86	0.95	0.46
$\gamma = 5$	4.16	5.25	6.87	9.44	3.37	4.42	5.98	8.47	0.77	0.80	0.84	0.89	0.58
$\gamma = 7.5$	4.91	6.32	8.51	12.36	4.09	5.46	7.61	11.38	0.79	0.81	0.84	0.88	0.65
$\beta = 0.96$													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.61	0.65	0.69	0.73	0.61	0.65	0.69	0.73	0.61	0.65	0.69	0.73	-
$\gamma = 1$	2.41	3.58	4.53	5.94	1.47	2.71	3.67	5.05	0.93	0.85	0.83	0.85	0.36
$\gamma = 2.5$	2.98	4.47	5.73	7.87	1.93	3.52	4.83	6.94	1.03	0.91	0.86	0.87	0.52
$\gamma = 5$	3.79	5.73	7.52	11.22	2.60	4.69	6.55	10.24	1.16	0.99	0.91	0.89	0.63
$\gamma = 7.5$	4.98	7.63	10.40	13.46	3.60	6.48	9.34	12.41	1.33	1.08	0.96	0.93	0.69
$\beta = 0.97$													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.61	0.65	0.69	0.73	0.61	0.65	0.69	0.73	0.61	0.65	0.69	0.73	-
$\gamma = 1$	3.22	3.99	5.08	6.69	1.96	2.58	3.48	4.83	1.24	1.38	1.55	1.77	0.48
$\gamma = 2.5$	4.20	5.24	6.74	9.01	3.17	4.13	5.52	7.65	0.99	1.06	1.15	1.26	0.61
$\gamma = 5$	4.97	6.31	8.31	11.58	4.04	5.32	7.25	10.42	0.90	0.94	0.99	1.05	0.69
$\gamma = 7.5$	5.61	7.27	9.92	14.79	4.69	6.30	8.90	13.67	0.88	0.91	0.94	0.99	0.73

Notes: The table displays the computed parameters for the decomposition of the welfare cost of total economic fluctuations using a robustness sample that excludes the interwar period. The numbers are obtained using equations (10) through (15) with the estimates shown in Table 9. All measures are in percentages of lifetime consumption. We also report an extra welfare cost measure, λ^{lit}, described in Appendix B.4. We report numbers for the relative degree of risk aversion $\gamma \in \{1, 2.5, 5, 7.5\}$, for the implied $\hat{\theta}_2$ along the grid for $\theta_1 \in \{0, 0.1, 0.2, 0.3\}$, and with $\beta \in \{0.95, 0.96, 0.97\}$ for the permanent shocks.

Table 11: Removing the interwar period - Welfare cost - ARIMA-BN

	$\beta = 0.95$												
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.60	0.64	0.68	0.72	0.60	0.64	0.68	0.72	0.60	0.64	0.68	0.72	-
$\gamma = 1$	3.73	4.62	5.88	7.75	2.21	2.92	3.95	5.51	1.48	1.65	1.86	2.13	0.60
$\gamma = 2.5$	5.39	6.76	8.74	11.81	4.01	5.26	7.10	9.96	1.32	1.42	1.53	1.68	0.83
$\gamma = 5$	6.76	8.72	11.81	17.26	5.45	7.34	10.31	15.59	1.24	1.29	1.36	1.44	0.96
$\gamma = 7.5$	7.97	10.79	15.96	29.41	6.68	9.42	14.48	27.68	1.21	1.25	1.29	1.35	1.01
	$\beta = 0.96$												
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.60	0.64	0.68	0.72	0.60	0.64	0.68	0.72	0.60	0.64	0.68	0.72	-
$\gamma = 1$	4.71	5.85	7.46	9.86	2.79	3.69	5.00	6.98	1.87	2.08	2.35	2.69	0.75
$\gamma = 2.5$	6.15	7.72	10.03	13.61	4.58	6.02	8.15	11.49	1.50	1.61	1.74	1.90	0.94
$\gamma = 5$	7.34	9.52	12.97	19.22	5.93	8.02	11.35	17.40	1.33	1.39	1.46	1.55	1.03
$\gamma = 7.5$	8.52	11.62	17.48	34.39	7.16	10.18	15.90	32.50	1.27	1.31	1.36	1.42	1.06
	$\beta = 0.97$												
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.60	0.64	0.68	0.72	0.60	0.64	0.68	0.72	0.60	0.64	0.68	0.72	-
$\gamma = 1$	6.38	7.94	10.15	13.46	3.76	4.99	6.77	9.49	2.52	2.81	3.16	3.63	1.01
$\gamma = 2.5$	7.14	8.99	11.73	16.03	5.32	7.02	9.54	13.54	1.73	1.85	2.00	2.19	1.08
$\gamma = 5$	8.03	10.46	14.37	21.65	6.50	8.82	12.60	19.65	1.44	1.50	1.58	1.67	1.11
$\gamma = 7.5$	9.15	12.58	19.32	42.06	7.70	11.05	17.64	39.96	1.34	1.38	1.43	1.50	1.12

Notes: The table displays the computed parameters for the decomposition of the welfare cost of total economic fluctuations using a robustness sample that excludes the interwar period. The numbers are obtained using equations (18) through (20) with the estimates shown in Table 9. All measures are in percentages of lifetime consumption. We also report an extra welfare cost measure, λ^{lit}, described in Appendix B.4. We report numbers for the relative degree of risk aversion $\gamma \in \{1, 2.5, 5, 7.5\}$, for the implied $\hat{\theta}_2$ along the grid for $\theta_1 \in \{0, 0.1, 0.2, 0.3\}$, and with $\beta \in \{0.95, 0.96, 0.97\}$ for the ARIMA-BN process.

F.2 Using Barro and Ursúa (2010) Sample Data

We now show our results using Barro and Ursúa (2010) data only. Table 12 shows the regression estimates and Tables 13, 14, and 15 show the calculation of the welfare costs for all types of shocks and for different β's.

There are some differences with the results of the analysis in the main text that require qualification. The first occurs in the case of transitory shocks, where we observe a lower volatility in the post-war period when compared to the augmented sample. This leads to a significant increase in the span of stabilization power and thus a higher welfare benefit of ongoing policies. Since this is not our preferred structure for the shocks, we understand that it does not make a substantial difference for our main findings.

The second difference is that for the case of ARIMA-BN shocks, the sample is also well-modeled by an ARIMA(0,1,1), yielding two different sets of welfare costs for these shocks, shown in Tables 14 and 15. If we compare the results for the ARIMA(1,1,0) depicted in Table 14, which is the one equivalent to the analysis in our main text, we observe that the total cost of economic fluctuations is roughly 1 percentage point smaller, a difference that is not substantial for the main message of our analysis.

Table 12: Barro and Ursúa (2010) Data - Estimation

Transitory shocks				Permanent shocks				ARIMA			
Estimated parameters				Estimated parameters				Estimated parameters			
	$\hat{\pi}_0$	$\hat{\pi}_1$	$\hat{\sigma}_u^2$		$\hat{\pi}_0$	$\hat{\pi}_1$	$\hat{\sigma}_u^2$		$\hat{\pi}_0$	$\hat{\phi}_1$	$\hat{\sigma}_u^2$
1835 - 1946	7.7417	0.0109	0.0065	1835 - 1946	0.0113	-	0.0021	1835 - 1946	0.0113	-	0.0021
	(.0156)	(.0002)			(.0043)				(.0043)		
1947 - 2009	6.4230	0.0233	0.0010	1947 - 2009	0.0210	-	0.0003	1947 - 2009	0.0207	0.2674	0.0003
	(.0330)	(.0002)			(.0023)				(.0030)	(.1316)	
Implied parameters				Implied parameters				Implied parameters			
θ_1	$\hat{\theta}_2$	$\hat{\alpha}_1$	$\hat{\sigma}_\varepsilon^2$	θ_1	$\hat{\theta}_2$	$\hat{\alpha}_1$	$\hat{\sigma}_\varepsilon^2$	θ_1	$\hat{\theta}_2$	$\hat{\alpha}_1$	$\hat{\sigma}_\varepsilon^2$
0.00	0.5976	0.0236	0.0065	0.00	0.6058	0.0217	0.0021	0.00	0.6223	0.0292	0.0021
0.10	0.6378	0.0236	0.0080	0.10	0.6453	0.0217	0.0026	0.10	0.6600	0.0293	0.0026
0.20	0.6781	0.0236	0.0101	0.20	0.6847	0.0218	0.0032	0.20	0.6978	0.0293	0.0032
0.30	0.7183	0.0236	0.0132	0.30	0.7241	0.0218	0.0042	0.30	0.7356	0.0294	0.0042

Notes: The table displays the estimated parameters obtained by running regression (22) for transitory shocks and regression (25) for permanent shocks. The time series used is the Barro and Ursúa (2010) data. The implied parameters are obtained using the formulas described in the text and equation (30).

Table 13: Barro and Ursúa (2008) Data - Welfare Cost

Transitory shocks													
	λ^T				λ^B				λ^R				λ
$\hat{\theta}_2$	0.60	0.64	0.68	0.72	0.60	0.64	0.68	0.72	0.60	0.64	0.68	0.72	-
$\gamma = 1$	0.32	0.40	0.51	0.66	0.19	0.25	0.34	0.47	0.13	0.14	0.16	0.19	0.05
$\gamma = 2.5$	0.81	1.00	1.27	1.66	0.60	0.78	1.03	1.39	0.21	0.22	0.24	0.26	0.13
$\gamma = 5$	1.63	2.01	2.55	3.35	1.28	1.65	2.17	2.94	0.34	0.35	0.37	0.40	0.26
$\gamma = 7.5$	2.45	3.04	3.86	5.07	1.97	2.54	3.34	4.52	0.47	0.49	0.50	0.53	0.39
Permanent shocks													
$\beta = 0.95$													
	λ^T				λ^B				λ^R				λ
$\hat{\theta}_2$	0.54	0.58	0.63	0.68	0.54	0.58	0.63	0.68	0.54	0.58	0.63	0.68	-
$\gamma = 1$	2.10	2.60	3.30	4.33	1.27	1.67	2.25	3.11	0.82	0.91	1.03	1.18	0.32
$\gamma = 2.5$	3.37	4.20	5.38	7.16	2.53	3.29	4.39	6.05	0.82	0.88	0.95	1.04	0.51
$\gamma = 5$	4.47	5.65	7.40	10.22	3.60	4.73	6.43	9.15	0.84	0.87	0.92	0.98	0.64
$\gamma = 7.5$	5.28	6.81	9.23	13.55	4.38	5.87	8.23	12.47	0.86	0.89	0.92	0.96	0.72
$\beta = 0.96$													
	λ^T				λ^B				λ^R				λ
$\hat{\theta}_2$	0.54	0.58	0.63	0.68	0.54	0.58	0.63	0.68	0.54	0.58	0.63	0.68	-
$\gamma = 1$	2.63	3.86	4.86	5.62	1.58	2.90	3.92	4.67	1.03	0.94	0.90	0.91	0.40
$\gamma = 2.5$	3.25	4.81	6.16	7.28	2.09	3.77	5.16	6.29	1.14	1.00	0.94	0.94	0.58
$\gamma = 5$	4.14	6.18	8.10	9.93	2.81	5.04	7.04	8.87	1.29	1.08	0.99	0.97	0.69
$\gamma = 7.5$	5.44	8.24	11.25	14.78	3.91	6.97	10.09	13.62	1.47	1.19	1.05	1.02	0.76
$\beta = 0.97$													
	λ^T				λ^B				λ^R				λ
$\hat{\theta}_2$	0.54	0.58	0.63	0.68	0.54	0.58	0.63	0.68	0.54	0.58	0.63	0.68	-
$\gamma = 1$	3.52	4.36	5.55	7.31	2.12	2.79	3.77	5.24	1.37	1.53	1.72	1.97	0.54
$\gamma = 2.5$	4.51	5.63	7.25	9.71	3.38	4.42	5.92	8.22	1.09	1.16	1.26	1.38	0.67
$\gamma = 5$	5.32	6.76	8.94	12.51	4.30	5.68	7.78	11.24	0.98	1.02	1.08	1.14	0.75
$\gamma = 7.5$	6.02	7.82	10.75	16.26	5.01	6.77	9.62	15.02	0.96	0.99	1.03	1.08	0.80

Notes: The table displays the computed parameters for the decomposition of the welfare cost of total economic fluctuations using a robustness sample with only the original Barro and Ursúa (2010) data. The numbers are obtained using equations (10) through (15) with the estimates shown in Table 12. All of the entries are in percentages of lifetime consumption. We also report an extra welfare cost measure, λ, described in Appendix B.4. We report numbers for the relative degree of risk aversion $\gamma \in \{1, 2.5, 5.10\}$, for the implied $\hat{\theta}_2$ along the grid for $\theta_1 \in \{0, 0.1, 0.2, 0.3\}$, and with $\beta \in \{0.95, 0.96, 0.97\}$ for the permanent shocks.

Table 14: Barro and Ursúa (2008) Data - Welfare Cost - ARIMA(1,1,0)

$\beta = 0.95$													
	λ^T				λ^B				λ^R				λ
$\hat{\theta}_2$	0.62	0.66	0.70	0.74	0.62	0.66	0.70	0.74	0.62	0.66	0.70	0.74	-
$\gamma = 1$	3.81	4.73	6.02	7.93	2.36	3.10	4.16	5.78	1.42	1.58	1.78	2.04	0.54
$\gamma = 2.5$	5.39	6.76	8.75	11.82	4.13	5.39	7.24	10.12	1.21	1.30	1.41	1.55	0.73
$\gamma = 5$	6.69	8.62	11.65	16.96	5.52	7.38	10.31	15.47	1.11	1.16	1.22	1.30	0.83
$\gamma = 7.5$	7.84	10.56	15.48	27.76	6.70	9.35	14.18	26.25	1.07	1.10	1.15	1.20	0.88
$\beta = 0.96$													
	λ^T				λ^B				λ^R				λ
$\hat{\theta}_2$	0.62	0.66	0.70	0.74	0.62	0.66	0.70	0.74	0.62	0.66	0.70	0.74	-
$\gamma = 1$	4.82	5.98	7.63	10.09	2.97	3.91	5.27	7.32	1.79	2.00	2.25	2.57	0.67
$\gamma = 2.5$	6.13	7.70	9.99	13.56	4.70	6.14	8.27	11.62	1.37	1.47	1.59	1.74	0.82
$\gamma = 5$	7.24	9.36	12.73	18.77	5.98	8.03	11.28	17.15	1.19	1.24	1.30	1.39	0.89
$\gamma = 7.5$	8.34	11.31	16.84	31.84	7.14	10.04	15.45	30.21	1.12	1.16	1.20	1.26	0.92
$\beta = 0.97$													
	λ^T				λ^B				λ^R				λ
$\hat{\theta}_2$	0.62	0.66	0.70	0.74	0.62	0.66	0.70	0.74	0.62	0.66	0.70	0.74	-
$\gamma = 1$	6.52	8.11	10.38	13.77	4.01	5.28	7.13	9.95	2.42	2.69	3.03	3.47	0.91
$\gamma = 2.5$	7.08	8.92	11.62	15.88	5.43	7.12	9.63	13.61	1.57	1.68	1.82	2.00	0.94
$\gamma = 5$	7.88	10.24	14.03	21.00	6.52	8.79	12.45	19.22	1.27	1.33	1.40	1.49	0.96
$\gamma = 7.5$	8.91	12.19	18.46	37.69	7.64	10.84	16.99	35.89	1.18	1.22	1.26	1.32	0.97

Notes: The table displays the computed parameters for the decomposition of the welfare cost of total economic fluctuations using a robustness sample with only the original Barro and Ursúa (2010) data. The numbers are obtained using equations (10) through (15) with the estimates shown in Table 12. All of the entries are in percentages of lifetime consumption. We also report an extra welfare cost measure, λ, described in Appendix B.4. We report numbers for the relative degree of risk aversion $\gamma \in \{1, 2.5, 5.10\}$, for the implied $\hat{\theta}_2$ along the grid for $\theta_1 \in \{0, 0.1, 0.2, 0.3\}$, and with $\beta \in \{0.95, 0.96, 0.97\}$ for the permanent shocks.

Table 15: Barro and Ursúa (2008) Data - Welfare Cost - ARIMA(0,1,1)

$\beta = 0.95$													
	λ^T				λ^B				λ^R				λ
$\hat{\theta}_2$	0.62	0.66	0.70	0.74	0.62	0.66	0.70	0.74	0.62	0.66	0.70	0.74	-
$\gamma = 1$	3.30	4.09	5.21	6.85	2.04	2.68	3.61	5.00	1.23	1.37	1.54	1.77	0.46
$\gamma = 2.5$	4.70	5.87	7.57	10.16	3.59	4.68	6.26	8.69	1.06	1.14	1.23	1.35	0.64
$\gamma = 5$	5.83	7.45	9.92	14.09	4.79	6.35	8.74	12.78	0.99	1.03	1.09	1.16	0.74
$\gamma = 7.5$	6.76	8.91	12.54	20.01	5.74	7.83	11.38	18.72	0.97	1.00	1.04	1.09	0.80
$\beta = 0.96$													
	λ^T				λ^B				λ^R				λ
$\hat{\theta}_2$	0.62	0.66	0.70	0.74	0.62	0.66	0.70	0.74	0.62	0.66	0.70	0.74	-
$\gamma = 1$	4.16	5.16	6.57	8.67	2.57	3.38	4.54	6.31	1.55	1.72	1.94	2.22	0.58
$\gamma = 2.5$	5.31	6.65	8.60	11.60	4.07	5.31	7.11	9.92	1.20	1.28	1.39	1.52	0.72
$\gamma = 5$	6.27	8.04	10.76	15.42	5.16	6.86	9.49	14.01	1.05	1.10	1.16	1.23	0.79
$\gamma = 7.5$	7.15	9.47	13.45	21.98	6.08	8.34	12.23	20.61	1.01	1.05	1.09	1.14	0.83
$\beta = 0.97$													
	λ^T				λ^B				λ^R				λ
$\hat{\theta}_2$	0.62	0.66	0.70	0.74	0.62	0.66	0.70	0.74	0.62	0.66	0.70	0.74	-
$\gamma = 1$	5.61	6.96	8.90	11.77	3.45	4.55	6.13	8.53	2.08	2.31	2.61	2.98	0.78
$\gamma = 2.5$	6.11	7.67	9.95	13.48	4.68	6.12	8.23	11.54	1.37	1.46	1.58	1.74	0.82
$\gamma = 5$	6.79	8.73	11.76	17.02	5.60	7.46	10.39	15.49	1.13	1.18	1.24	1.32	0.85
$\gamma = 7.5$	7.59	10.11	14.50	24.43	6.46	8.91	13.21	22.96	1.06	1.10	1.14	1.19	0.87

Notes: The table displays the computed parameters for the decomposition of the welfare cost of total economic fluctuations using a robustness sample with only the original Barro and Ursúa (2010) data. The numbers are obtained using equations (18) through (15) with the estimates shown in Table 12. All of the entries are in percentages of lifetime consumption. We also report an extra welfare cost measure, λ, described in Appendix B.4. We report numbers for the relative degree of risk aversion $\gamma \in \{1, 2.5, 5.10\}$, for the implied $\hat{\theta}_2$ along the grid for $\theta_1 \in \{0, 0.1, 0.2, 0.3\}$, and with $\beta \in \{0.95, 0.96, 0.97\}$ for the permanent shocks.