

Adjustment Costs, Financial Constraints and the Persistence of Misallocation in China

PRELIMINARY AND INCOMPLETE

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Abstract

Using firm level data in the years 2000-2013 on all Chinese publicly traded manufacturing firms divided among private, state-owned and privatized enterprises, we document the presence of misallocation in the Chinese economy. We then develop a dynamic investment model with physical and financial frictions where firms make their financing decisions in the presence of operating, equity issuance, adjustment and financing costs where the latter are captured by a collateral constraint that allows lenders to discriminate among firms according to their ownership. Using the firm level data, we estimate the structural parameters of the model which is able to generate TFP losses relative to the efficient allocation of 59% and dispersion in TFPR that closely matches the data. We explore a counterfactual analysis comparing the levels of aggregate productivity to the efficient level when adjustment costs are removed and find that 35% of the misallocation generated by the model in baseline economy disappears without the presence of such costs. Finally, we observe that adjustment costs and a tight financing restriction interact to amplify misallocation losses by spreading firms' size distribution.

Keywords: Misallocation, adjustment costs, corporate finance

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1 Introduction

In the economics literature, differences in per-capita income across countries have been historically explained using differences in capital deepening, human capital and technology levels. Recently a literature has analyzed how these resources are allocated across different producers within a country and found large differences of allocative efficiency between developing and developed countries, which implies aggregate productivity levels far below optimal. Misallocation occurs when the most productive firms use a small share of the economy’s inputs, and thus operate at a small scale, while less productive firms control a large share of inputs and produce a large share of the output. If inputs were reallocated from the least to the most productive firms, output and measured TFP would rise. [Hsieh and Klenow \[2009\]](#) provide evidence of substantial efficiency losses in the Chinese manufacturing sector due to misallocation of productive inputs, in the order of 86% of total factor productivity (TFP) if it were perfectly allocated, and 40% when compared with the efficiency of the United States.

These efficiency losses are typically mapped to distortions in the economy, either on the input or the product markets that make it difficult for the most productive firms to grow and capture a large share of the market. [Hsieh and Song \[2015\]](#) find that in the input side most of these distortions seem to arise from the capital side, as there is more variation in the average product of capital than in labor in the Chinese economy.

In this paper we argue that a large proportion of this variation can be explained in a model that faces dynamic investment decisions and adjustment costs, and as such are technological constraints that would be faced by a social planner, implying that a large proportion of the TFP losses measured in the data should not necessarily be considered to be policy failures, but as the natural outcome of an economy where heterogeneous firms that are advancing in their life cycle and responding optimally to productivity shocks.

We gather firm level data on all Chinese publicly traded manufacturing firms in the years 2000-2013 and document the presence of misallocation within the companies studied. In light of such evidence, we build a model of the firm that faces dynamic investment and financing decisions in the presence of financial constraints and adjustment costs, where lenders can potentially treat firms differently according to their ownership. We use the firm level data to estimate the structural parameters of the model. Most recent models of misallocation from the industrial sector are not able to generate large TFP losses stemming from misallocation [[Midrigan and Xu, 2014](#), [Gopinath et al., 2015](#)], but our model is able to generate dispersion in TFPR that is close to the data. We use this framework to explore counterfactual exercises where we compare the levels of aggregate productivity to the efficient level when the firms face adjustment costs, and when these are removed. We find that 35% of the misallocation generated by the model in baseline economy disappears once we remove convex adjustment costs from the model.

The channel that introduces this dispersion is simple: the level of adjustment costs that are necessary to match the investment moments in the data induce the path of capital accumulation to vary more, being more dependent on the particular history of productivity shocks that a firm faces. When these adjustment costs are removed, more firms grow faster to their optimal level ending up looking more alike, reducing the degree of variation in marginal products, the main measure of misallocation. The key point is that adjustment costs are unavoidable and would be faced by a social planner.

Finally, we explore how do these adjustment costs interact with financial constraints, here modeled as collateral constraints and establish that adjustment costs amplify the TFP losses generated through a type of selection: when collateral constraints are tight and adjustment costs high, more firms are small due to either exiting when they get a bad shock, or by simply slowing down capital accumulation enough. When these collateral constraints are loosened, increasing adjustment costs does not spread out the distribution as much as firms can use debt to grow more quickly.

Related Literature: This paper is a contribution to the growing literature of misallocation, with early contributions by [Restuccia and Rogerson \[2008\]](#) and [Hsieh and Klenow \[2009\]](#), and the research of misallocation associated with financial frictions. [Midrigan and Xu \[2014\]](#), [Hsieh and Song \[2015\]](#), and [Brandt et al. \[2013\]](#) agree in finding substantial TFP losses due to misallocation in China. [Moll \[2014\]](#) shows that parametrization choices of models of misallocation can have large results in the measured aggregate TFP losses. This paper is most closely related to [Asker et al. \[2015\]](#), who make a similar point in remarking that much of the variation of average products can be generated by adjustment costs as firms respond optimally to productivity shocks. This paper provides a model with which the actual aggregate productivity losses can be quantified. [Buera and Shin \[2013\]](#) study the effects of financial frictions on the persistence of misallocation after reforms that distort production, and the length of time they take to unwind; suggesting that gradual, rather than one off reforms must have been the case in developing countries as their model grows to the steady state too quickly.

This paper is related to the literature of dynamic models of corporate finance, which [Strebulaev and Whited \[2012\]](#) survey. Our model draws from the work of [Gomes \[2001\]](#), [Hennessy and Whited \[2007\]](#) and [Li et al. \[2016\]](#). [Cooper and Haltiwanger \[2006\]](#) make an important contribution highlighting the importance of capital adjustment costs in being able to match firm investment moments in this literature, and they highlight the importance of including both convex and non-convex costs of adjustment which are not very large, but statistically and economically significant.

The rest of the paper proceeds as follows, Section 2 describes the data and presents the main misallocation results. Section 3 describes the model and Section 4 provides functional forms and gives a brief description of the estimation procedures. Section 5 highlights the results and provides intuition for them, and finally Section 6 gives concluding remarks.

2 Misallocation and Adjustment Cost Facts

We use Balance Sheet, Cash Flow, Income Statements, and shareholder information on all publicly traded firms in China for the period 2000-2013 in the Shanghai and Shenzhen exchanges from the CSMAR database. To determine the ownership of each type of firm we follow a methodology similar to [Hsieh and Song \[2015\]](#), where we define a firm as SOE if: it has more than 50% shares officially listed as state owned, or if the largest shareholder is the state or a state owned entity. We only keep manufacturing firms, which leaves me with a sample with 13,608 annual observations of 1644 firms. As [Hsieh and Song \[2015\]](#) point out, it will be important to also keep track separately of firms which at one point during the sample were SOE, but have since been privatized, as these tend to be fall in between SOE and private firms for most observable characteristics.

To measure misallocation we use the framework introduced by [Hsieh and Klenow \[2009\]](#), which uses unspecified wedges to capture all distortions that drive the economy away from a perfect allocation of resources. Suppose the economy is populated by N different sectors, and that in each sector s in period t there are M_{ts} monopolistically competitive firms each producing differentiated foods $y_{t si}$. These are combined into the aggregate sectoral good Y_{ts} with a Constant Elasticity of Substitution (CES) aggregator with elasticity ν :

$$Y_{ts} = \left[\sum_{i=1}^{M_{ts}} y_{t si}^{\frac{\nu-1}{\nu}} \right]^{\frac{\nu}{\nu-1}}$$

Each firm uses a Cobb-Douglas production function with idiosyncratic productivity $z_{t si}$ and labor share $1 - \alpha_s$ that is sector specific:

$$y_{t si} = z_{t si} k_{t si}^{\alpha_s} l_{t si}^{1-\alpha_s}.$$

The firm observes demand and chooses prices, capital and labor to maximize profits $\pi_{t si}$:

$$\pi_{t si} = \max_{p_{t si}, k_{t si}, l_{t si}} p_{t si}(y_{t si})y_{t si} - (1 + \tau_{t si}^l)w l_{t si} - (1 + \tau_{t si}^k)(r + \delta)k_{t si}$$

where $p(y_{si}) = P_s(y_{si}/Y_s)^{\frac{-1}{\nu}}$ is the usual downward sloping demand curve that results from consumer optimization under monopolistic competition, τ_{si}^l and τ_{si}^k are firm specific wedges that affect the cost of labor and capital respectively. With these we attempt to capture all possible restrictions, subsidies and any other distortions that affect the firm's marginal product of capital and labor. First order conditions are the usual, prescribing that the marginal benefits, here titled as marginal revenue products of labor and capital, equal their marginal costs:

$$MRPL_{t si} \equiv (1 - \alpha_s) \left(\frac{\nu - 1}{\nu} \right) \frac{p_{t si} y_{t si}}{l_{t si}} = (1 + \tau_{t si}^l)w \quad (1)$$

$$MRPK_{t si} \equiv \alpha_s \left(\frac{\nu - 1}{\nu} \right) \frac{p_{t si} y_{t si}}{k_{t si}} = (1 + \tau_{t si}^k)(r + \delta) \quad (2)$$

Note that the marginal costs include the wedges that prohibit the firm from reaching its optimal size, as these wedges raise (if there is a distortion), or lower (if there is a subsidy) marginal costs, rendering firms unable to as optimality prescribes equalize costs and benefits. A firm that has difficulties in accessing credit, for example, would have a high $\tau_{t si}^k$, implying that they have a high marginal revenue product of capital and that it can't borrow to increase their capital stock. Following the literature we define a firm's total factor revenue productivity, $TFPR_{t si}$, as:

$$TFPR_{t si} \equiv p_{t si} z_{t si}$$

A result of consumer optimization is that if there are no distortions present in the economy, $TFPR_{t_{si}}$ should be equalized across firms, since every firm faces the same problem they would choose the same allocations after controlling for productivity. If you are a very productive firm, which maps to having a high $z_{t_{si}}$, you should capture a large share of the market, which under monopolistic competition leads to a lower price $p_{t_{si}}$. When $TFPR_{t_{si}}$ is high relative to other firms, it implies that the firm is very productive but small, and when it is low, that the firm is large but unproductive. Thus, if this variable presents a large degree of variation, this is evidence of abundant misallocation.

To measure $TFPR_{t_{si}}$ we use:

$$TFPR_{t_{si}} \equiv p_{t_{si}} z_{t_{si}} = \frac{p_{t_{si}} y_{t_{si}}}{k_{t_{si}}^{\alpha_s} l_{t_{si}}^{1-\alpha_s}}, \quad (3)$$

Where $p_{t_{si}} y_{t_{si}}$ will be a measure of value added¹, $k_{t_{si}}$ will be fixed assets computed with a perpetual inventory method, and $l_{t_{si}}$ the labor input measured by cash paid to and on behalf of employees. For this analysis only firms with positive value added are kept. The sector specific labor shares $1 - \alpha_s$ are computed from aggregate data and are detailed in Appendix C. In Figure 1 we plot the distribution of log deviations of $TFPR$ from industry means² by current ownership. Summary statistics are reported in Table 1.

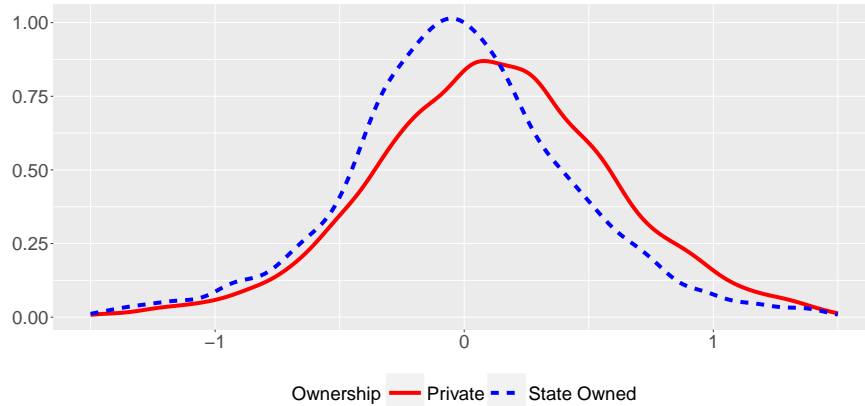


Figure 1: Distribution of $\log(TFPR)$ Deviations from industry means

First, the presence of misallocation is clear, as there is wide variation in TFPR within industries. Second, on average, SOE tend to be larger than their productivity warrants, with mean deviation of -.048, as they tend to have low TFPR relative to industry average; while privately owned firms tend to be smaller than their productivity warrants with mean deviation of .095. A Kolmogorov-Smirnoff test of whether the distribution for private firms lies below (and hence has more mass on higher values) than the SOE distribution has a p-value of 2.2×10^{-16} . The same pattern is observed when we consider Privatized SOE separately, with privatized being slightly less large for their productivity than SOE, while presenting more variation of TFPR than both other types of firms.

¹Value added will be defined as the sum of Operating Profits (EBIT), cash paid for and on behalf employees and depreciation.

²The statistic is $\log(TFPR_{t_{si}}/\overline{TFPR}_s)$, where \overline{TFPR}_s is an industry average of $TFPR_{t_{si}}$, defined in Appendix F.

Table 1: Statistics of $TFPR$ deviations from industry means

	N	Mean	Median	S.D.	75 - 25	90 - 10
<i>By Ownership</i>						
SOE	6,680	-.048	-.042	.614	.559	1.204
Private	6,052	.096	.105	.624	.643	1.286
<i>By Status</i>						
SOE	5,663	-.053	-.040	.594	.549	1.183
Private	4,972	.136	.138	.578	.623	1.240
Privatized SOE	2,097	-.048	-.049	.749	.658	1.430
<i>All Groups</i>						
All Years	12,732	.015	.025	.623	.613	1.250
H-K Data 2001	108,702			.68	.88	1.71
H-K Data 2005	211,304			.63	.82	1.59

Notes: The table reports summary statistics of $\log(TFPR_{t,si}/\overline{TFPR}_s)$ over all firms. S.D is standard deviation, 75-25 is the interquartile range and 90-10 is the difference between the 90th and 10th percentiles. Industries are weighted by share of value added. Only firms with positive value added are kept.

By conducting the same counterfactual study as [Hsieh and Klenow \[2009\]](#) of eliminating all distortions in the economy we find smaller TFP gains, 19% compared to 86% in their paper. This result is expected, given that they use a survey that covers all firms above a revenue threshold, and we only observe publicly traded firms, hence our sample presents much less variation in TFPR than theirs does, as can be seen in the bottom panel of [Table 1](#). The difference between the 75th and 25th percentiles of TFPR in my data is .613, and for them it is between .82 and .88, and the same happens with the 90th and 10th percentiles. Furthermore, if we don't trim the tails of the distortions and productivity measures, the losses increase to 52%.

To consider the misallocation of capital and labor separately, we plot the standard deviation of log marginal revenue products of capital and labor separately in [Figure 2](#), which, according to equations (1)-(2) should capture the extent of distortions in these two inputs. If there is significant variation in each one of these, there would be evidence for distortions of that input, modeled as variation in the respective wedge. The data points out that there is more distortion of capital than labor, a fact consistent with [Hsieh and Song \[2015\]](#) who document that the labor productivity of SOE had largely converged with that of the private firms by 2007 while capital productivity in SOE remained smaller. They argue that this reflects lower redundant employment in SOE as they shed 3.6 million workers from 1998-2007, while the higher capital productivity of private firms is likely driven by their difficulty in accessing funding for capital investment. It is interesting to note that distortions peak around the crisis era, but have since fallen back to pre-crisis level, potentially an effect of the stimulus policies undertaken during the crisis being targeted sub-optimally, but having since been undone.

In this paper we focus on the distortions to capital exclusively, and will not allow any channel that distorts labor choices. Given that it's typically modeled as a static choice, we assume it has been optimized away.

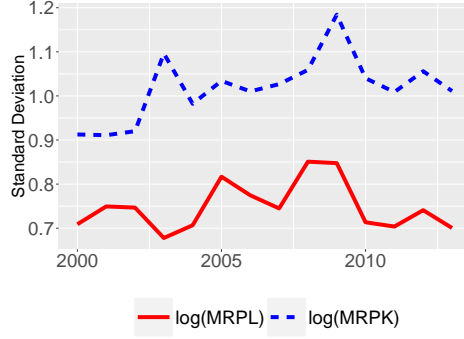


Figure 2: Path of dispersion of $\log(MRPK)$ and $\log(MRPL)$

We now provide some reduced form evidence of the importance of adjustment costs in amplifying the spread of MRPK in the data. If it is the case that they are significant, then when a firm receives a productivity shock, this should amplify the dispersion of marginal products. As output, the numerator of MRPK in equation (2) will rise, but the denominator, the capital stock will take a long time to grow in order to equalize. This is different from the effect of the distortions, as they have an effect on the levels of MRPK, rather than a dynamic effect.

To test if this is the case, we estimate innovations to each firm’s $TFPR$ process using a dynamic panel model with the methodology of [Arellano and Bond \[1991\]](#). The model is:

$$\log(TFPR_{t+1si}) = \rho \log(TFPR_{tcsi}) + v_i + \sum_{j=2001}^{2013} \gamma_{tj} + \varepsilon_{tcsi} \quad (4)$$

This results in $\hat{\rho} = .286$. With these estimated innovations $\hat{\varepsilon}_{tcsi}$ we estimate the following regression as in [Asker et al. \[2015\]](#):

$$\log(MRPK_{tcsi}) = \beta_0 + \beta_1 \hat{\varepsilon}_{tcsi} + \beta_2 \log(k_{tcsi}) + \beta_3 \log(l_{tcsi}) + \beta_4 \log(TFPR_{t-1si}) + u_{tcsi}. \quad (5)$$

These regressions ask the following question: faced with the same capital and lagged productivity levels, do firms who receive different productivity shocks end up with different marginal products of capital? If adjustment costs were irrelevant, firms would invest/disinvest optimally and $MRPK$ would remain constant across firms, rendering $\beta_1 = 0$. On the other hand, if it takes time to adjust the capital stock following a shock, a high TFPR innovation today would raise a firm’s marginal revenue product for a long time, implying $\beta_1 > 0$. The results are in [Table 2](#).

For all specifications of the regression $\beta_1 > 0$ at the .01% level, implying that firms don’t optimally adjust their capital stock immediately after receiving a productivity shock, even after controlling for industry and individual productivity, and their current input levels. The coefficient implies that a 1% innovation in TFPR increases the marginal revenue product of capital about .5%, which imply that adjustment costs are present in economically and statistically significant amounts. In [Appendix E](#) more lags of $\hat{\varepsilon}_{tcsi}$ are added as a robustness check for timing inconsistencies, and we find that at both one and

Table 2: Dispersion of $MRPK$ and Productivity Shocks

	<i>Dependent variable:</i>			
	$\log(MRPK)$			
	(1)	(2)	(3)	(4)
$\log(TFPR)$ Innovation	0.467*** (0.032)	0.469*** (0.032)	0.703*** (0.018)	0.596*** (0.023)
Capital Stock	-0.600*** (0.017)	-0.606*** (0.018)	-0.775*** (0.014)	-0.933*** (0.024)
Labor Input	0.623*** (0.016)	0.641*** (0.017)	0.736*** (0.014)	0.791*** (0.035)
Lagged $\log(TFPR)$	0.158*** (0.019)	0.164*** (0.019)	0.674*** (0.022)	0.440*** (0.030)
Constant	-1.928*** (0.170)	-2.074*** (0.170)	-0.186* (0.108)	1.530*** (0.391)
Year F.E.	No	Yes	Yes	Yes
Industry F.E.	No	No	Yes	No
Individual F.E.	No	No	No	Yes
Observations	11,122	11,122	11,122	11,122
R^2	0.298	0.307	0.834	0.885

*p<0.1; **p<0.05; ***p<0.01

We report the results of the regression of equation (5) for different sets of controls, where the $\log(TFPR)$ Innovation is given as the estimated residual of the model of equation (4), the Capital Stock is given by fixed assets and $TFPR$ is defined as in equation (3). In parentheses are heretoskedasticity robus standard errors.

two lags of TFPR innovations the coefficient is positive and significant, providing further evidence that marginal products take a long time to adjust.

3 Model

This section presents the model used to study the investment decisions of firms which generates endogenous sources of variation of marginal products. The model follows closely a modified version of [Li et al. \[2016\]](#), which is a dynamic investment model with a contracting problem, but with the addition of an exit decision modeled as in [Clementi and Palazzo \[2015\]](#), with monopolistic competition.

There is a continuum of monopolistically competitive producers making differentiated goods y_t , whose objective is to maximize the expected value of the firm. Within the continuum of firms, there will be three types, denoted $i \in \{a, b, c\}$, respectively private, privatized and SOE firms. For clarity any parameters that depend on the type of the firm will have an i index. Each firm enters a period with a productivity shock $z \in \mathcal{Z}$, debt $b \in \mathcal{B}$ and capital $k \in \mathcal{K}$, where $\mathcal{K} \times \mathcal{B} \times \mathcal{Z} \subset \mathbb{R}^3$. The firm faces a demand curve given by $p(y) = y^{-1/\nu}$. Once it enters the period the firm produces using its capital stock according and the technological shock, which follows a Markov process with transition function $Q^i(dz', z)$ which satisfies the Feller property. Firm output is given by the production function F :

$$y(k, z, i) = F(k, z, i)$$

Operating profits are given by

$$\pi(k, z, i) = p(y(k, z, i)) y(k, z, i)$$

After production, the firm chooses whether to pay a random fixed cost c_f with distribution function C_f^i to continue operating. If it chooses to exit, then it pays off its debts $(1 + r_b)b$ and returns to shareholders the produced output and the depreciated value of capital. If the firm decides to pay the fixed cost, it must choose how much capital to have next period k' and how much to borrow b' at rate $1 + r$ from a deep pockets bank, subject to a collateral constraint:

$$(1 + r)b' \leq \theta^i(1 - \delta)k'$$

[Li et al. \[2016\]](#) derive this constraint as a reformulation of an enforcement constraint on a long term debt contract, but restrict equity distributions to be positive. The firm discounts with β , pays adjustment costs of capital according to the function $A^i(k, k')$, and if it issues negative distributions, it must pay equity issuance costs which are captured in the function $\Phi^i(\cdot)$. The capital and borrowing decisions imply how large are equity distributions e , to be defined below.

3.1 Recursive Problem

The recursive problem for a firm with type i can be written down as follows:

$$V(k, b, z, i) = \int_{c_f} \max_{\text{exit, operate}} \{V^E(k, b, z, i), V^O(k, b, z, i) - c_f\} dC^i(c_f) \quad (6)$$

The value of exiting is distributing profits and depreciated capital net of debts:

$$V^E(k, b, z, i) = \pi(k, z, i) + (1 - \delta)k - (1 + r)b. \quad (7)$$

The value of operation after paying the fixed cost is given by:

$$V^O(k, b, z, i) = \max_{k', b'} \Phi^i(e(k, k', b, b', z, i)) + \beta \int_{z'} V(k', b', z', i) Q^i(dz', z) \quad (8)$$

equity distributions are given by:

$$e(k, k', b, b', z, i) = \pi(k, z, i) + (1 - \delta)k - (1 + r)b - k' - A^i(k, k') + b' \quad (9)$$

subject to

$$(1 + r)b' \leq \theta^i(1 - \delta)k' \quad (10)$$

The equity issuance costs function Φ^i charges a constant fraction ϕ^i of the raised value, paid only if distributions are negative:

$$\Phi^i(e) = e + \mathbf{1}_{[e < 0]} \phi^i e. \quad (11)$$

Since the fixed cost enters linearly, the exit decision is given by a cutoff rule where there exists a realization of the fixed cost $c_f^*(k, b, z, i)$ that renders the firm indifferent between exiting and staying, and thus the firm will exit with probability $\mathbb{P}[c_f \geq c_f^*(k, b, z, i)] = x^i(k, b, z)$, and stay with complementary probability. By a law of large numbers, at each state the measure of firms that choose to stay will pay an expected fixed cost of

$$\mathbb{E}[c_f | c_f \leq c_f^*(k, b, z, i)] \equiv c_f^e(k, b, z, i)$$

The solution to this problem is given by policies $k'(k, b, z, i)$, $x^i(k, b, z)$, $b'(k, b, z, i)$, $e(k, b, z, i)$ and a value function $V(k, b, z, i)$. We can rewrite the value function as:

$$V(k, b, z, i) = x^i(k, b, z)V^E(k, b, z) + (1 - x^i(k, b, z))[V^O(k, b, z, i) - c_f^e(k, b, z, i)]$$

3.2 Aggregation

Firms are born with initial capital endowment k_0^i and zero debt, and a shock drawn from the stationary distribution of $Q^i(dz', z)$, denoted as $\bar{Q}^i(dz)$. Any firm that exits is replaced with a newborn firm. Let $\mu^i(k, b, z)$ denote the mass of firms in the state $(k, b, z) \in \mathcal{K} \times \mathcal{B} \times \mathcal{Z}$, for any for any subset $S = (K, B, Z) \in \Sigma(\mathcal{K}) \times \Sigma(\mathcal{B}) \times \Sigma(\mathcal{Z})$, where $\Sigma(X)$ denotes the minimum sigma algebra for set X . For compactness denote $s = (k, b, z)$. The law of motion for the measure of firms of type i can be characterized in the following way, for the sets that contain the state at which firms are born, if $(k_0^i, 0) \in (K, B)$

$$\mu'^i(S) = \int_{\mathcal{K}} \int_{\mathcal{B}} \int_{\mathcal{Z}} \int_{\mathcal{Z}} \chi^i(s, S) [\mu^i(ds)(1 - x^i(s))] Q^i(dz', z) + \int_{\mathcal{Z}} M^i \bar{Q}^i(dz) \quad (12)$$

for cases when it is not, $(k_0^i, 0) \notin (K, B)$

$$\mu'^i(S) = \int_{\mathcal{K}} \int_{\mathcal{B}} \int_{\mathcal{Z}} \int_{\mathcal{Z}} \chi^i(s, S) [\mu^i(ds)(1 - x^i(s))] Q^i(dz', z). \quad (13)$$

Where χ^i is an indicator function of whether the policy functions points to a specific state:

$$\chi^i(s, S) = \begin{cases} 1 & \text{if } (k'(s, i), b'(s, i)) \in (K, B) \\ 0 & \text{otherwise} \end{cases} \quad (14)$$

and M^i is the mass of exitors, whom are just reborn at initial capital and zero debt:

$$M^i = \int_K \int_Q \int_Z x^i(s) \mu^i(ds) \quad (15)$$

This equation takes into account the exit, investment and borrowing decisions of firms to keep track of the state of each firm of type i across the whole distribution. To obtain the aggregate measure μ , add the three measures together $\sum_{i,s} \mu^i(s) = \mu(s)$.

Since the equilibrium definition will be a stationary industry equilibrium, it will be the case that $\mu' = \mu = \mu^*$ so that μ defines the mass of firms in a stationary equilibrium.

Using this stationary distribution, we define aggregate variables in the following way, aggregate debt is given by

$$B = \sum_{i \in \{a,b,c\}} \int_S b \mu^i(ds) \quad (16)$$

aggregate capital,

$$K = \sum_{i \in \{a,b,c\}} \int_S k \mu^i(ds) \quad (17)$$

aggregate output,

$$Y = \left[\sum_{i \in \{a,b,c\}} \int_S y(k, z, i)^{\frac{\nu-1}{\nu}} \mu^i(ds) \right]^{\frac{\nu}{\nu-1}}, \quad (18)$$

aggregate TFP will be measured as

$$TFP \equiv \frac{Y}{K} = \frac{\overline{TFPR}}{P} = \left[\sum_{i=a,b,c} \int_S \left(z \frac{\overline{TFPR}}{TFPR(s,i)} \right)^{\nu-1} \mu^i(s) \right]^{\frac{1}{\nu-1}} \quad (19)$$

where $\overline{TFPR} \equiv PY/K$ is aggregate revenue productivity and $TFPR(s, i) = p(y(s, i))z$ is defined as in the data section. Finally, given that efficient allocation of resources is achieved when all firms equalize $TFPR(s, i) = \overline{TFPR}$, so that if capital is perfectly allocated, the efficient level of TFP is

$$TFP^e = \left[\sum_{i=a,b,c} \int_S z^{\nu-1} \mu^i(s) \right]^{1/(\nu-1)}. \quad (20)$$

These equations are derived in Appendix G.

3.3 Stationary Industry Equilibrium

Definition : A stationary industry equilibrium is a set of policies for each type of firm $k'(k, b, z, i)$, $b'(k, b, z, i)$, $x^i(k, b, z)$, a value function $V(k, b, z, i)$ and three firm measures $\mu = (\mu^a, \mu^b, \mu^c)$ such that

- Firm decision rules and value function solve each firm's problem (6)-(11);
- The measures μ satisfy the law of motion (12)-(15) with $\mu' = \mu$.

Proposition 1 *There exists a stationary industry equilibrium.*

Proof See Appendix B

3.4 Sources of Misallocation

To clarify what are the sources of misallocation in the model, we show the optimality condition with respect to capital. Suppose that the Lagrange multiplier of the collateral constraint is $\lambda\beta$, and denote δ_h as the partial derivative with respect to h , omitting the arguments of the policy functions. The first order condition of next period capital, k' , in the case of positive distributions (for simplicity), is given by

$$\begin{aligned} 1 + \delta_{k'} A^i(k, k') &= \beta \mathbb{E}_{z'} [\delta_{k'} V(k', b', z', i) + \lambda \theta^i (1 - \delta) | z] \\ 1 + \delta_{k'} A^i(k, k') &= \beta \mathbb{E}_{z'} \left[\alpha^i \left(\frac{\nu - 1}{\nu} \right) p(z' k'^{\alpha^i}) z' k'^{\alpha^i - 1} + (1 - \delta) \right. \\ &\quad \left. + X(k', b', z', i) + \lambda \theta^i (1 - \delta) | z \right] \end{aligned}$$

Rearranging the discounting and depreciation terms,

$$\begin{aligned} \frac{1 + \delta_{k'} A^i(k, k')}{\beta} + \delta - 1 &= \mathbb{E}_{z'} \left[\alpha^i \left(\frac{\nu - 1}{\nu} \right) \frac{p' y'}{k'} + X(k', b', z', i) | z \right] + \lambda \theta^i (1 - \delta) \\ \frac{1 + \delta_{k'} A^i(k, k')}{\beta} + \delta - 1 &= \mathbb{E}_{z'} [MRPK' + X(k', b', z', i) | z] + \lambda \theta^i (1 - \delta) \end{aligned}$$

The left hand side of this equation is the user cost of capital, and the right hand side is marginal revenue product of capital, plus the shadow value of relaxing the collateral constraint. The term $X(k', b', z', i)$ captures the marginal returns related with the exit decision and adjustment costs:

$$\begin{aligned} X(k', b', z', i) &= \delta_{k'} x^i(\cdot) [k'' + A^i(k', k'') - b' + \mathbf{1}_{[e < 0]} \phi^i e' + c_f^e - \beta V''] \\ &\quad + x^i(\cdot) [\delta_{k'} A^i(k', k'') + \mathbf{1}_{[e < 0]} \phi^i + \delta_{k'} c_f^e] \end{aligned}$$

In the model variation in the measured marginal revenue product of capital MRPK, which is defined here identically as for the data analysis in equation (2), is generated by variation of the adjustment costs

firms are facing, equity issuance costs and the exit decision through the X term, risks in investment, and the collateral constraint, which affects the choice of capital at its shadow value λ . Implicit in this equation is also the initial level of capital k_0^i , which can be interpreted as IPO size of the firm, which affects where along distribution of marginal products of capital the firms are located in the stationary distribution.

The two effects that this paper highlights are the contribution of adjustment costs to generating dispersion in MRPK through X and the time-to-build technology that requires that capital purchased today to be productive tomorrow. We study how do collateral constraints (θ^i) and adjustment costs interact to make the growth process slower, and the TFP losses from misallocation larger.

4 Estimation and Identification

Now that the equilibrium has been defined, this section describes the functional forms chosen for the model, the estimation procedure and the identification of parameters.

4.1 Functional Forms

Firms have access to a Cobb-Douglas production function, where the capital share will be the same for all types of firms:

$$F(z, k, l) = zk^{\alpha^i}$$

The stochastic process for the technology shock is given by

$$\log(z') = (1 - \rho^i)m_z^i + \rho^i \log(z) + \sigma_z^i \epsilon$$

where $\epsilon \sim N(0, 1)$. This process allows for firms of different ownership types to have productivity with different means, variances and persistence levels. The distribution of the random fixed costs C_f^i will be lognormal with parameters $(m_{c_f}^i, \sigma_{c_f}^{i2})$. The functional form of adjustment costs of capital feature fixed and convex costs, as is typical in the investment literature, and is key to fitting the investment moments in the data (see [DeAngelo et al. \[2011\]](#), [Cooper and Haltiwanger \[2006\]](#)),

$$A^i(k, k') = \mathbf{1}_{[k' \neq k]} \kappa_0^i k + \frac{\kappa_1^i k}{2} \left(\frac{k' - (1 - \delta)k}{k} \right)^2.$$

4.2 Method of Simulated Moments & Identification

We estimate most of the structural parameters of the model using the method of simulated moments (MSM), however some of the parameters will have to be estimated separately or fixed from outside data. The discount factor is set to the average real 1-year lending rate in China for 2000-2013, which was 5.6%, so that $\beta = \frac{1}{1.056}$. As is typical of the investment literature, in order for the firm's problem to be well defined even in the long run, it is required that $\beta < \frac{1}{1+r}$, basically to render the firm less patient than it's lenders. This is typically achieved if taxes are included in the model, but since we don't explicitly model taxes, and this gap is very difficult to identify in the data, we use the corporate tax rate of 25% to set $r = (1 - .25) \times 0.056 = 0.042$.³ This rate implies that agents don't have an incentive to pay down their debts once they reach their optimal size, and thus are either newborn with zero debt, or at their constraints at all times.

The depreciation rate is set at $\delta = .09$ to match the ratio of depreciation to fixed assets in the data. The elasticity of substitution is $\nu = 3$ to match the TFPR analysis of Section 2. The means of the stochastic processes for the technology shock are estimated using the method of Wooldridge [2009], which is a GMM refinement of Olley and Pakes [1996]. We average the resulting log productivity measures across firm types, and express them as ratios to SOE log productivity so that $m_z^c = \log(1)$, private firms productivity is $m_z^a = \log(2.2)$ and privatized productivity is $m_z^b = \log(1.2)$. Given that we cannot identify in the data if a firm is delisted, closes or is taken private, we take the exit rates from the aggregate data of Hsieh and Song [2015], where the average exit rate from 1998-2007 was 12% for private firms and 13.2% for SOE. For privatized firms we take the average of those two.

The estimated parameters are $p = (\{\rho^i, \sigma_z^i, \theta^i, \omega^i, k_0^i, \kappa_0^i, \kappa_1^i, \alpha^i\}_{i \in \{a,b,c\}})$, where ω^i stands for the ratio of the absolute value of the mean to the standard deviation of the fixed cost. We express k_0^i as a percentage of the non-stochastic steady state level of capital. Given that the exit rate and equity issuance rates are almost uniquely dependent and monotone on the mean of the fixed cost $m_{c_f}^i$ and the equity issuance cost parameter ϕ^i respectively, for every solution of the model at parameters p use a bisection algorithm over these two to match the two moments very closely. The relative mass of each type of firm will be given by the share of aggregate output each produced within the sample, so that aggregate quantities match the relative contributions of each type. This results in SOE being 66%, Private firms 23% and Privatized SOE 11%.

The idea of MSM is to estimate the parameters of an average firm by choosing them so that the average moments that result from a simulation are close to those from the data. In a model where the stationary distribution of firms is obtained, one can avoid simulation error by computing population moments in the model by using the distribution and policy functions of the firms.

We match the mean and standard deviation of equity issuance, and the fraction of periods with positive issuance⁴. We also match the mean, standard deviation and AR(1) coefficients of investment, operating profits and debt, where debt is defined as the sum of short term debt, long term debt and bonds outstanding. All moments will be measured relative to fixed assets, as the model does not include any investment in intangibles or working capital. My model counterparts will be $(k' - (1 - \delta)k)/k$ for

³Given that if the firm borrows one dollar and pays r of interest on it, they get to deduct r dollars from their taxable income, which pays rate tax , this renders the effective interest rate as $(1 - tax) \times r$.

⁴This frequency is normalized by the number of periods a firm is in the sample.

investment, b/k for debt, profits will be π/k , and equity issuance will be $\mathbf{1}_{[e<0]}e/k$.

In MSM identification comes from carefully choosing moments that are sensitive to the structural parameters of the model. A parameter is well identified by the MSM estimator if it has a monotonic relationship with a data moment [Strebulaev and Whited, 2012]. It is also important that the moments are related to the variables that will be affected in the counterfactual. In the model, the standard deviation and autocorrelation of profits are most informative about ρ^i and σ_z^i . A higher collateral constraint parameter θ^i corresponds to larger levels of mean debt while the correlation of debt and investment is useful for placing debt within the pecking order of sources of financing, mostly through k_0^i .

Mean investment and equity issuance depend on the cost parameter ϕ^i and the initial level of capital k_0^i . The standard deviation of investment informs the curvature of the production function α^i as firms won't invest as aggressively as a result of a productivity shock if the firm presents large decreasing returns to scale. The adjustment costs parameters, κ_0^i and κ_1^i are related to the standard deviation of investment and to its autocorrelation. The computational method to solve the model and technical details about how to solve the model and on MSM are given in Appendix A.

5 Results

In Table 4 we present the data and simulated moments that resulted from the estimation. The model is able to match the data fairly well. The moments that are the most important for the counterfactuals to be studied, those relating to investment, are well captured, as the mean, standard deviation and autocorrelation of investment moments are very close to the data. The profit moments, which relate to production are also well captured, as also is the debt level. The model has difficulty achieving equity issuance levels that match the data and at the same time reasonable profitability and investment levels, which occurs because the firms would be on average smaller when equity issuance is high, inducing the ratios of investment and profits to assets to be much larger. Given that those moments are better measured in the data (the variance-covariance matrix from which the weights are derived presents a much smaller variance for mean investment and profits), the estimator prefers to match these two over equity issuance.

Table 3: Parameter Estimates

Type	θ^i	ρ^i	σ_z^i	ω^i	k_0^i	κ_0^i	κ_1^i	α^i	χ^2
Private	0.637	0.834	0.387	23.786	0.004	0.000	1.811	0.976	5.366
SOE	0.731	0.875	0.465	17.899	0.014	0.033	2.031	0.971	4.314
Privatized	0.838	0.861	0.477	12.129	0.026	0.023	1.264	0.959	5.274

Notes: The table reports the parameter values estimated with Method of Simulated Moments. χ^2 is the statistic for the J test of overidentifying restrictions. The 95th percentile of the distribution is 11.07.

Table 3 contains the point estimates of parameter values that result from the estimation. The debt to capital ratios are monotonic in the debt levels that they match, a typical feature of models with

collateral constraints. Private firms start smaller than SOE, which in turn start smaller than privatized firms, inducing the patterns in equity issuance observed in the data, but at much smaller scales given the difficulties discussed before.

The model is fit well, as can be evidenced by the results of the J-test of over identifying restrictions, given by the last column of Table 3, where the test statistics for all three estimations are well to the left of the 95th quantile, failing to reject the null hypothesis that the model is not identified. Some non-targeted moments that the model also fits well are the time series moments of productivity, ρ^i and σ_z^i , which are very close to those that arise from the previously estimated productivity measures using the same dynamic panel regression as in equation (4). In the data, the autocorrelation coefficient has a value of .801, with standard error of the regression of .511, which are close to the values found across the firm types in Table 3.

Finally, to assess how closely the model captures measured misallocation relative to the data, it is useful to compare the variation in $\log(TFPR)$ from Table 1, and the aggregate results from the baseline parametrization in Table 5. The standard deviation was .62 while in the model it was .47, therefore the model does not generate the same degree of variation, and it can be seen that it is due to the model not having that many firms in the tails, as the log difference of 90th and 10th quantiles is 1.05, while in the data it was 1.25, yet the gap between the 75th and the 25th quantiles are very close. Given that the model is very parsimonious in how distortions from optimality are generated, this is to be expected.

The model is able to generate TFP losses relative to the efficient allocation of resources of 59%, in the range of those found by Hsieh and Klenow [2009] who determine they are 86%, much larger than those from the data used in this paper, which were only 19%, again, mostly due to a much smaller sample of more homogeneous firms.⁵ In the next counterfactual, we explore the contribution of the presence of adjustment costs to this large degree of inefficiencies.

⁵This low number is also due to following the same trimming procedure of Hsieh and Klenow [2009], where as if one does not trim the 1% tails of TFPR and the distortions, the TFP losses relative to the efficient are 52%, implying that the tails of this distribution are very important in determining the degree of misallocation generated.

Table 4: Method of Simulated Moments Results

Model	Moments	Private		SOE		Privatized	
		Data	Model	Data	Model	Data	Model
$\frac{k' - (1-\delta)k}{k}$	Mean Investment / Assets	0.341	0.226	0.202	0.118	0.166	0.112
	SD Investment / Assets	0.149	0.180	0.107	0.104	0.110	0.118
	AR(1) Investment / Assets	0.600	0.589	0.692	0.596	0.600	0.542
$\frac{\pi}{k}$	Mean Profits / Assets	0.309	0.267	0.150	0.204	0.102	0.179
	SD Profits / Assets	0.147	0.121	0.126	0.091	0.160	0.083
	AR(1) Profits / Assets	0.501	0.615	0.508	0.671	0.446	0.644
$\frac{b}{k}$	Mean Debt / Assets	0.476	0.490	0.671	0.554	0.774	0.640
	Correlation Debt and Investment	-0.036	-0.358	0.009	-0.178	0.039	-0.002
$\frac{1_{[e < 0]}}{k}$	Mean Equity Issuance / Assets	0.217	0.022	0.154	0.003	0.245	0.001
	SD Equity Issuance / Assets	0.417	0.081	0.079	0.025	0.062	0.009
	Frequency of Equity Issuance / Assets	0.779	0.226	0.157	0.023	0.133	0.042
	Correlation Equity and Investment	0.149	0.047	0.091	0.017	0.058	0.020

Notes: The table reports simulated and data moments for each firm type. The data comes from the CSMAR database and covers the years 2000-2013, all moments are relative to fixed assets, we trim 3% of the tails of each ratio before taking the statistics.

In order to clarify the effects of capital deepening and selection in the model, the fourth column of Table 5 shows the aggregate effects of greatly relaxing financial constraints, by increasing the collateral constraint parameter of all types of firms to $\theta^i = 2$. The main thing to notice is that measured TFP falls by 22%, which is the result of two forces: first, given that the production function of the firms presents decreasing returns to scale, and TFP is defined as Y/K , keeping other things constant, increasing the scale of the economy will lower the average marginal productivity, which is captured as lower TFP. Thus, while output is greatly increased, the quantity of capital required to produce it is more than proportionally larger, and measured TFP falls. Second, the average physical productivity of the surviving firms falls, as looser financial constraints allows more small and low productivity firms to survive. Due to this selection effect, the different distributions give different levels of efficient TFP, TFP^e in equation (20), so for the rest of the paper we present both TFP losses relative to the baseline, and to the counterfactual efficient levels of TFP.

Table 5: Aggregate Results

	Data	Baseline	$\theta^i = 2$
Aggregate TFP		2.41	1.87
TFP Losses	19%	59%	85%
TFP Losses To Baseline		59%	106%
Output (Y)		763	1946
Capital (K)		315	1040
Debt (B)		192	1806
SD $\log(TFPR)$.62	.47	.58
75-25 $\log(TFPR)$.61	.60	.62
90-10 $\log(TFPR)$	1.25	1.05	1.41
Exit Rate	12%	13%	6%

The table contains the aggregate variables of the economy, computed as in equations (16)-(20). The first column names the aggregate moments, the second column contains the relevant moments from the data, the third column has the baseline parametrization and the fourth column contains a counterfactual where the collateral constraint parameter is set for all firms at $\theta^i = 2$. The TFP losses are measured as $100(TFP^e/TFP - 1)$, where TFP^e can be either the model's, or the one from the baseline economy and represent how much TFP would rise if it were perfectly allocated. 75-25 and 90-10 stand for the difference of the quantiles of the stationary distribution of $\log(TFPR)$.

5.1 Counterfactuals

We present results of the counterfactual exercises. For all of these only the parameters detailed are changed, so there is no new bisection performed, as can be seen by the changes in exit rates, an important channel through which adjustment costs and financial constraints interact to affect TFP losses.

The counterfactual exercise of Table 6 removes the adjustment costs by setting either $\kappa_0^i = 0$, $\kappa_1^i = 0$, or both, and presents the same aggregate values, divided by those of the baseline economy for clarity. From the second column it can be seen that removing fixed adjustment costs is able to both increase the scale of the economy and marginally increase TFP, and from the third column, removing convex adjustment costs increased TFP by 10% and brought the economy much closer to the efficient level, by 20 percentage points. When both are removed TFP increases by only 6%, and this case is the one that minimizes variation of TFPR at .34, approximately half of the variation in the data. However, removing fixed adjustment costs makes it easier for SOE and privatized firms to capture a larger share of the market, and because these are the least productive types of firms, aggregate productivity falls.

This key result implies that for this model to generate sizable TFP losses relative to an efficient TFP level, it requires a technological constraint would also be encountered by a social planner, and has been established to be necessary in order to accurately capture the timing of investment decisions found in the data, and thus is unavoidable. This implies that a large proportion of the measured misallocation by the literature may not be caused by bad policies, but is an inherent part of the firm life cycle as they respond to productivity shocks along their capital accumulation process.

Note that the increase in output is fivefold, and it occurs because without these adjustment costs, the firms optimally choose to grow very quickly, and are at their optimal size in one period by issuing large amounts of equity. Deviations of optimality only occur when the productivity shock changes, or when they exit because they received a large fixed cost shock. Thus, these firms spend most of the time in their optimal size, rather than being distributed along the path to it as in the baseline, and this distribution produces much more output. This fact is robust to larger values of the equity issuance costs.

Table 6: Counterfactual: Removing Adjustment Costs (Relative to Baseline)

	Baseline	Remove Fixed AC	Remove Convex AC	Remove Both
Aggregate TFP	1	1.01	1.10	1.06
TFP Losses	59%	55%	38%	37%
TFP Losses to Baseline	59%	57%	44%	50%
Output (Y)	1	1.17	5.29	4.61
Capital (K)	1	1.16	4.78	4.36
Debt (B)	1	1.17	4.87	4.44
SD $\log(TFPR)$.47	.47	.39	.34
Exit Rate	13%	12%	9%	6%

The table contains results of counterfactual exercises, with the aggregate variables computed as in equations (16)-(20). The first column names the aggregate moments, all of which will be expressed a ratio to the baseline economy. The second column contains the baseline economy, the third column the counterfactual of removing fixed adjustment costs, the third of removing convex adjustment costs and the fourth of removing both types of adjustment costs. The TFP losses are measured as $100(TFP^e/TFP - 1)$, where TFP^e can be either the model's, or the one from the baseline economy and represent how much TFP would rise if it were perfectly allocated. 75-25 and 90-10 stand for the difference of the quantiles of the stationary distribution of $\log(TFPR)$.

In order to study the interaction of adjustment costs and financial constraints we first present in Table 7 the aggregate results of equalizing collateral parameters θ^i and initial capital levels k_0^i at their value added weighted means. We map k_0^i to IPO size and can be thought of as a financial parameter. In the data SOE firms have a mean log assets in their first year of being listed of 21.01, while private of 20.75, so the estimated differences of k_0^i are broadly in line with the data. As can be seen in this table equalizing these constraints does not increase TFP by a large amount, just about 2%. However, this process improves selection as the TFP losses relative to the baseline efficient TFP are smaller than those of the counterfactual efficient TFP, indicating that the surviving firms are on average more productive when θ^i is equalized. Thus, in the presence of adjustment costs, the misallocation due to different financial treatment across firm types are about 20% of those that arise from adjustment costs.

Table 7: Counterfactual: Equalizing Financial Constraints (Relative to Baseline)

	Baseline	$\theta^i = .72$	$k_0^i = .01$	Both
Aggregate TFP	1	1.02	1.01	1.02
TFP Losses	59%	58%	56%	59%
TFP Losses to Baseline	59%	55%	57%	56%
Output (Y)	1	.74	1.02	.96
Capital (K)	1	.72	1.01	.94
Debt (B)	1	.61	1	.92
SD $\log(TFPR)$.47	.49	.44	.47
Exit Rate	13%	15%	12%	15%

The table contains results of counterfactual exercises, with the aggregate variables computed as in equations (16)-(20). The first column names the aggregate moments, all of which will be expressed a ratio to the baseline economy. The second, third and fourth columns contain the aggregate quantities that result from equalizing $\theta^i = .72$, $k_0^i = .01$, and from equalizing both simultaneously respectively. The TFP losses are measured as $100(TFP^e/TFP - 1)$, where TFP^e can be either the model's, or the one from the baseline economy and represent how much TFP would rise if it were perfectly allocated. 75-25 and 90-10 stand for the difference of the quantiles of the stationary distribution of $\log(TFPR)$.

In Table 8 we compare the interaction of the collateral constraints and adjustment costs channels by comparing how much do TFP losses increase as the level of adjustment costs increases for tight financial constraints ($\theta^i = .5$), and loose financial constraints ($\theta^i = 1$). We solve the model after setting the level of adjustment costs low: 10% of the baseline adjustment costs parameters, medium at 50% and high at 200% of the adjustment costs levels.

The effects of relaxing collateral constraints are consistent with the capital deepening and decreasing returns to scale technology, as discussed in the beginning of this section: for all levels of adjustment costs, relaxing financial constraints reduces aggregate TFP as a result of firms operating at a larger scale with decreasing returns to scale technology. What's important to highlight is how these magnitudes differ at different levels of collateral constraints.

If collateral constraints are tight, increasing the level of adjustment costs increases the TFP losses by 20 percentage points, whereas doing the same for loose financial constraints increases TFP losses by the smaller amount of 13 percentage points. The main channel is the fattening of the right tail: when financial constraints are tight and adjustment costs high, more firms exit relatively early and are born with a small level of capital, which is associated with high TFPR: the 90th quantile is .60 when constraints are tight and .53 when they are loose. In contrast, when financial constraints are loose, more of these firms survive, and bring aggregate TFP down. Thus, adjustment costs and collateral constraints interact to make the losses from misallocation larger: when constraints are tight, higher adjustment costs make the size distribution of firms more spread out as firms are kept small, yet if these are loose, occurs happens much faster.

Table 8: Counterfactual: Adjustment Costs Experiment (Relative to Baseline)

	$\theta = .5$			$\theta = 1$		
	Low	Mid	High	Low	Mid	High
Aggregate TFP	1.09	1.04	1.00	1.01	1.01	.97
TFP Losses	43%	55%	63%	52%	61%	65%
TFP Losses to Baseline	46%	52%	59%	58%	60%	64%
Output (Y)	2.55	1.37	.51	2.60	1.77	.48
Capital (K)	2.38	1.31	.51	2.57	1.79	.50
Debt (B)	1.64	.90	.31	3.64	2.50	.64
SD $\log(TFPR)$.40	.47	.53	.42	.49	.52
10th Q TFPR	.15	.14	.21	.14	.15	.22
25th Q TFPR	.21	.21	.22	.19	.19	.22
50th Q TFPR	.24	.27	.37	.22	.22	.30
75th Q TFPR	.28	.38	.48	.27	.34	.49
90th Q TFPR	.38	.45	.60	.37	.45	.53
Exit Rate	11%	13%	18%	12%	12%	16%

The table contains results of counterfactual exercises, with the aggregate variables computed as in equations (16)-(20). The first column names the aggregate moments, all of which will be expressed a ratio to the baseline economy, as well as the 10th, 25th, 50th, 75th and 90th quantiles of the distribution of TFPR. The second to seventh columns display these moments for different values of adjustment costs, which vary from Low (10% of each model's adjustment costs in the baseline estimation), Medium (50%) and High (200%), and the level of financial constraints.

6 Concluding Remarks

In this paper we study misallocation within the context of a dynamic investment model in order to measure how important are adjustment costs to misallocation as is measured by the variation in marginal products, the most common measure of the literature. We find that the small sample presents a large

degree of variation in these marginal products and that TFP losses from misallocation are large, similarly to studies done with a larger sample.

We build and estimate a dynamic investment model of a firm that faces adjustment costs and financial constraints using Chinese data and the method of simulated moments. This model is able to generate variation in marginal products and TFP losses from misallocation that are in the order of what was measured in the data.

Using the model we conduct several counterfactual exercises to examine if adjustment costs are a big source of the TFP losses found. To do this we remove adjustment costs from the model and conclude that 35% of the losses arise due to the presence of adjustment costs, while a negligible amount arises from financial constraints. This result provides evidence that much of the measured TFP losses may be unavoidable by a social planner, and thus efficient. Finally, we study how financial constraints and adjustment costs interact to worsen the allocation of resources, which occurs through both a selection effect (more unproductive firms survive), and a capital deepening effect.

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Appendix

A Details of the Computational Method

The model is solved globally for the policy and value functions using Value Function Iteration. We discretize the state space (k, b, z) by generating a grid for capital that ranges from 5×10^{-4} to 2 times the non-stochastic steady state value of capital. We generate log-spaced grids of size 60 for these two states. The stochastic process is discretized using Tauchen and Hussey's method, with 5 grid points. As mentioned in Section 4.2, for every solution with parameters $p_i = (\rho^i, \sigma_z^i, \theta^i, \omega^i, k_0^i, \kappa_0^i, \kappa_1^i, \alpha^i)$, we perform a bisection of the mean of the fixed cost parameter $m_{c_f}^i$ and the equity issuance cost parameter ϕ^i to get the exit rate and the average ratio of equity issuance to fixed assets to match the data for every firm type i .

The estimator is from Lee and Ingram [1991]. The goal is to estimate the parameter vector p_i using data vector $\{d_i\}$, where this contains for every firm i all observations T_i of the relevant data series. These vectors are assumed to be assumed to be *i.i.d* across i but there may be dependence within i . Let h map from the data to the moments, so this function takes the appropriate means and standard deviations, correlations and autocorrelation and frequency of positive equity issuance from the series $h(d_i)$. We estimate each firm's parameters independently. For the simulated data, given that we have the stationary distribution of the policy functions of the model it is not necessary to simulate the firms, as one can obtain any moment produced by the model from it. Denote the moments generated by parameter vector p_i by $h^m(p_i)$, using the stationary distributions μ^i , and define the sample moment vector as

$$g_i(x, p) = \frac{1}{N} \sum_{j=1}^N [h(d_j) - h^m(p_i)]$$

Then, for some random weight matrix \hat{W} which converges in probability to a deterministic positive definite matrix W the Simulated Method of Moments estimator is given by

$$\hat{p}_i = \arg \min_p g_i(x, p)' \hat{W} g_i(x, p)$$

The weight matrix \hat{W} is chosen to be for each firm type i the inverse of the covariance matrix of the moments. We first use a global optimization algorithm (DIRECT-L from the NLOpt library) and a local optimizer afterwards for more precise estimation within this minima (Sbplx). The programming language used are Julia and C for the modeling, and R for the data related sections.

B Proofs

Proof of Proposition 1. Similarly to Li et al. [2016], we use Theorem 9.6 in Stokey et al. [1989] to establish the existence of the industry equilibrium defined in Section 3.3. Theorem 9.8 establishes uniqueness of the policy functions. The proposition can be restated by defining an operator T in the space of bounded function as

$$T(V)(k, b, z, i) = x^i(k, b, z)V^E(k, b, z) + (1 - x^i(k, b, z))[V^O(k, b, z, i) - c_f^e(k, b, z, i)]$$

where

$$V^E(k, b, z, i) = \pi(k, z, i) + (1 - \delta)k - (1 + r)b,$$

$$V^O(k, b, z, i) = \max_{k', b'} \Phi^i(e(k, k', b, b', z, i)) + \beta \int_{z'} V(k', b', z', i) Q^i(dz', z),$$

and

$$e(k, k', b, b', z, i) = \pi(k, z, i) + (1 - \delta)k - (1 + r)b - k' - A^i(k, k') + b'$$

subject to

$$(1 + r)b' \leq \theta^i(1 - \delta)k'$$

Lemma 1 *Let $C(S)$ be the space of all bounded continuous functions, where $S = \mathcal{K} \times \mathcal{B} \times \mathcal{Z}$. The operator T defined above has an unique fixed point $V^* \in C(S)$ for all $v_0 \in C(S)$ initial guesses. (Theorem 9.6 of Stokey et al. [1989])*

Proof This is established in Stokey et al. [1989], we only have to show that Assumptions 9.4-9.7 hold:

- Assumption 9.4: $\mathcal{K} \times \mathcal{B}$ are convex Borel sets in \mathbb{R}^2 with Borel subsets $\Sigma(\mathcal{K}) \times \Sigma(\mathcal{B})$. These assumptions are easily satisfied if for example $\mathcal{K} = [0, \bar{k}]$ and $\mathcal{B} = [\underline{b}, \bar{b}]$. The firm would never choose a capital stock larger than $\bar{k} = \bar{z}\bar{k} + \bar{k}(1 - \delta)$ because otherwise profits are negative. The upper bound for the debt can then be derived from the collateral constraint $\bar{b} = \theta(1 - \delta)\bar{k}/(1 + r)$. The lower bound must also be well defined because given that all firms are born with zero debt and the firms have no incentive to save after they reach their steady state value of capital given that they discount at a higher rate than the debt returns: if it sacrifices one dollar in dividends today, it returns $1 + r$ tomorrow, but they discount this at rate β . Given the assumed value of r as detailed in Section 4.2, $\beta \times (1 + r) < 1$, so the firm prefers to set $b = 0$ to $b < 0$, so that $\underline{b} = 0$.

- Assumption 9.5: \mathcal{Z} is a compact (Borel) set in \mathbb{R} with its Borel subsets $\Sigma(\mathcal{Z})$, and the transition function Q on $(\mathcal{Z}, \Sigma(\mathcal{Z}))$ has the Feller property. This is trivially satisfied by the Log-Normal AR(1) process as it is non-negative, and its transition maps the any bounded and continuous function back into a bounded and continuous function.
- Assumption 9.6: The constraint correspondence $\Gamma(k, b, z) : \mathcal{K} \times \mathcal{B} \times \mathcal{Z} \rightarrow \mathcal{K} \times \mathcal{B}$ given by:

$$\Gamma(k, b, z) = \{(k', b') | \theta k'(1 - \delta) \geq b'\}$$

is non-empty, compact-valued and continuous.

1. Non-empty: $k' = 0$ and $b' = 0$ belong to $\Gamma(k, b, z)$, thus non-empty.
2. Compact-valued: Pick any sequence $\{k'_n, b'_n\} \in \Gamma(k, b, z)$ for any z such that it converges, $\{k'_n, b'_n\} \rightarrow (k'_c, b'_c)$. Given that the constraint is linear, it must be the case that $\theta k'_c(1 - \delta) \geq b'_c$, independent of the z . $\Gamma(k, b, z)$ is closed and compact valued (contains all of its limit points).
3. To show that $\Gamma(k, b, z)$ is continuous we will show that it is both upper hemi-continuous and lower hemi-continuous. Since we have shown it is nonempty and bounded (given the definitions in Assumption 9.4), Theorem 3.4 gives that it is upper hemi-continuous. Pick any sequence $\{k'_n, b'_n, z'_n\}$, such that it converges, $\{k'_n, b'_n, z'_n\} \rightarrow (k'_c, b'_c, z'_c)$. To prove lower hemi-continuity one must show that for every $(f'_c, g'_c) \in \Gamma(k'_c, b'_c, z'_c)$ there exists $N \geq 1$ and a sequence $(f'_n, g'_n) \in \Gamma(k'_n, b'_n, z'_n)$ that is convergent $(f'_n, g'_n) \rightarrow (f'_c, g'_c) \forall n \geq N$. Given that $(k'_c, b'_c) \in \Gamma(k'_c, b'_c, z'_c)$. Convergence of the initial sequence means that for all $\{\delta_i > 0 | i \in \{k, b, z\}\}$ there exists some $\{N_i \in \mathbb{Z} | i \in \{k, b, z\}\}$ where for all $n > N_i$, $|k'_n - k'_c| < \delta_k$, $|b'_n - b'_c| < \delta_b$ and $|z'_n - z'_c| < \delta_z$. Let $N = \max\{N_k, N_b, N_z\}$ and $(f'_n, g'_n) = (a_k k'_n, a_b b'_n)$, where $a_k = f'_c/k'_c$ and $a_b = g'_c/b'_c$ then it is the case that $(f'_n, g'_n) \rightarrow (a_k k'_c, a_b b'_c) = (f'_c, g'_c)$ for all $n \geq N$.

- Assumption 9.7: $\beta \in (0, 1)$ and the function $F(k, b, z, k', b')$ is bounded and continuous,

$$F(k, b, z, k', b') = \pi(k, z) + (1 - \delta)k - (1 + r)b + (1 - x(k, b, z))[k' - A(k, k') + b' - c_f^e(k, b, z) + \mathbf{1}_{[e < 0]} \phi e.]$$

given that $\mathcal{K} \times \mathcal{B}$ is bounded, and all the component functions are continuous, this is satisfied.

Lemma 2 *Suppose $S = \mathcal{K} \times \mathcal{B} \times \mathcal{Z}$, Γ , Q , F and β satisfy Assumptions 9.4-9.7. If they additionally satisfy 9.10 and 9.11 then the value function is strictly concave and the optimal policies are unique and continuous. (Theorem 9.8 of [Stokey et al. \[1989\]](#)).*

Proof • Assumption 9.10: Given z , the F function satisfies, for all $t \in (0, 1)$

$$\begin{aligned} & F [tk_0 + (1 - t)k_1, tb_0 + (1 - t)b_1, z, tk'_0 + (1 - t)k'_1, tb'_0 + (1 - t)b'_1] \\ & \geq tF [k_0, b_0, z_0, k'_0, b'_0] + (1 - t)F [k_1, b_1, z_1, k'_1, b'_1] \end{aligned}$$

And strictly if any of the variables are different. This holds given that the production function has decreasing returns to scale, even when taking into account the pricing function. The only concerns are the equity issuance cost, but given that it is linear, it is weakly concave; and the exit decision which is a value along a log-normal CDF, and changes monotonically and continuously in the same direction of the elements contained in the brackets it multiplies

- Assumption 9.11: For all $z \in \mathcal{Z}$ and all $(k'_n, b'_n) \in \mathcal{K} \times \mathcal{B}$ for $n = 1, 2$, if $y_n \in \Gamma(k'_n, b'_n, z)$, then $ty_n + (1-t)y_n \in \Gamma(tk'_1 + (1-t)k'_2, tb'_1 + (1-t)b'_2, z)$ for all $t \in [0, 1]$. Given that the constraint is linear this holds trivially.

C Industry Labor Share

To measure the labor share of each industry for the *TFPR* analysis we use a similar definition as [Hsieh and Song \[2015\]](#), where it is defined as

$$\text{labor share} = \frac{\text{labor income}}{\text{labor income} + \text{total profit} + \text{depreciation} + \text{value added tax}}.$$

Ideally, this should be equal to aggregate labor share of 50% [[Chang et al.](#)] if you had a full sample of firms, but due to accounting and statistical discrepancies from reported data this is not the case in the full sample of firms that they use, and is not the case in our restricted sample either. The discrepancy arises from firms reporting only wages as part of payroll and a discrepancy between reported income and reported value added.

We obtain industrial sector Labor Income data from University of Michigan's China Data Center; Depreciation, Value Added Tax and Total Profits from the relevant for each year's China Statistical Yearbook. We obtain this data for all the manufacturing sectors for 2005-2008. As expected, the average labor share for my sample is 32%, less than the aggregate share of 50%, so we inflate labor income by a constant factor across all years and sectors, increasing by .0001 until the average labor share is exactly 50%.

D Data Comments

We use data on all manufacturing firms from 2000-2013 in the CSMAR database. Manufacturing firms are those that begin with the letter "C" in the 2012 CRSC Industry Code. The precise variable items from the CSMAR database are:

- Fixed Assets: A001212000
- Short Term Debt: A002101000
- Long Term Debt: A002201000
- Bonds: A002203000
- Dividends: A002115000
- Total Assets: A001000000

- Profits (Operating Profits): B001300000
- Investment: C002006000
- Proceeds from issuing shares: C003001000
- Payroll: C001020000
- Depreciation: D000103000
- Date Firm Established: ESTBDT
- Monthly Market Capitalization: MSMVTTL
- Tobin's Q is measured using the average market capitalization for a firm over each year, subtracting inventories from this amount, and dividing by total firm assets.
- The proceeds from issuing shares variable has missing data, but is coded erroneously as hapenning in the beginning of the following year as confirmed by comparing with a secondary source for the data, the ChinaScope database. This is corrected for the periods for which there is data in that entry. Some firms erroneously code their payroll/labor income data and fixed assets as negative, and this is also corrected.

E Different Lags of TFPR Innovations

In this appendix we present the results of adding one and two more lags of TFPR innovations to the regression equation 5, which shows that the result is robust to longer delays between the period of when the innovation is first observed, and when the capital is successfully adjusted.

Table 9: Dispersion of *MRPK* and Productivity Shocks with One Lag of Innovations

	<i>Dependent variable:</i>			
	log(<i>MRPK</i>)			
	(1)	(2)	(3)	(4)
log(<i>TFPR</i>) Innovation	0.641*** (0.032)	0.641*** (0.032)	0.845*** (0.015)	0.788*** (0.023)
log(<i>TFPR</i>) Innovation in $t - 1$	0.380*** (0.031)	0.377*** (0.031)	0.253*** (0.010)	0.309*** (0.014)
Capital Stock	-0.496*** (0.013)	-0.507*** (0.013)	-0.680*** (0.010)	-0.708*** (0.028)
Labor Input	0.583*** (0.014)	0.604*** (0.014)	0.698*** (0.013)	0.706*** (0.043)
Lagged log(<i>TFPR</i>)	0.197*** (0.019)	0.200*** (0.019)	0.755*** (0.024)	0.553*** (0.044)
Constant	-3.361*** (0.167)	-3.409*** (0.168)	-1.328*** (0.084)	-1.068*** (0.407)
Year F.E.	No	Yes	Yes	Yes
Industry F.E.	No	No	Yes	No
Individual F.E.	No	No	No	Yes
Observations	9,635	9,635	9,635	9,635
R ²	0.295	0.302	0.874	0.907

Note:

*p<0.1; **p<0.05; ***p<0.01

We report the results of the regression of equation (5) for different sets of controls, where the log(*TFPR*) Innovations are given as the estimated residual of the model of equation (4), the Capital Stock is given by fixed assets and *TFPR* is defined as in equation (3). In parentheses are heretoskedasticity robus standard errors.

Table 10: Dispersion of *MRPK* and Productivity Shocks with Two Lags of Innovations

	<i>Dependent variable:</i>			
	$\log(MRPK)$			
	(1)	(2)	(3)	(4)
$\log(TFPR)$ Innovation	0.683*** (0.033)	0.682*** (0.033)	0.861*** (0.016)	0.798*** (0.024)
$\log(TFPR)$ Innovation in $t - 1$	0.529*** (0.038)	0.525*** (0.039)	0.283*** (0.015)	0.404*** (0.024)
$\log(TFPR)$ Innovation in $t - 2$	0.274*** (0.030)	0.270*** (0.030)	0.061*** (0.015)	0.160*** (0.022)
Capital Stock	-0.474*** (0.014)	-0.485*** (0.014)	-0.676*** (0.012)	-0.710*** (0.035)
Labor Input	0.584*** (0.015)	0.605*** (0.016)	0.691*** (0.014)	0.712*** (0.051)
Lagged $\log(TFPR)$	0.161*** (0.023)	0.165*** (0.023)	0.761*** (0.030)	0.490*** (0.055)
Constant	-3.846*** (0.181)	-3.909*** (0.182)	-1.294*** (0.089)	-1.235*** (0.456)
Year F.E.	No	Yes	Yes	Yes
Industry F.E.	No	No	Yes	No
Individual F.E.	No	No	No	Yes
Observations	8,171	8,171	8,171	8,171
R ²	0.310	0.317	0.872	0.905

Note:

*p<0.1; **p<0.05; ***p<0.01

We report the results of the regression of equation (5) for different sets of controls, where the $\log(TFPR)$ Innovations are given as the estimated residual of the model of equation (4), the Capital Stock is given by fixed assets and *TFPR* is defined as in equation (3). In parentheses are heretoskedasticity robus standard errors.

F Industry TFPR

Industry level *TFPR* appears is used in order to compare which firms are too small relative to their productivity and which ones are too large in the data. Define industry average marginal revenue products as:

$$\begin{aligned}\overline{MRPK}_s &= \left[\sum_{i=1}^{M_s} \frac{1}{MRPK_{si}} \frac{p_{si}y_{si}}{P_s Y_s} \right]^{-1} = \left[\sum_{i=1}^{M_s} \frac{1}{(r + \delta)(1 + \tau_{si}^k)} \frac{p_{si}y_{si}}{P_s Y_s} \right]^{-1} \\ \overline{MRPL}_s &= \left[\sum_{i=1}^{M_s} \frac{1}{MRPL_{si}} \frac{p_{si}y_{si}}{P_s Y_s} \right]^{-1} = \left[\sum_{i=1}^{M_s} \frac{1}{w(1 + \tau_{si}^l)} \frac{p_{si}y_{si}}{P_s Y_s} \right]^{-1}\end{aligned}$$

where Y_s is as defined in the main text and $P_s = \left[\sum_{i=1}^{M_s} p_{si}^{1-\nu} \right]^{\frac{1}{1-\nu}}$ is the industry level price index. In the static problem, labor demand is given by:

$$l_{si} = (1 - \alpha_s) \left(\frac{\nu - 1}{\nu} \right) \frac{p_{si}y_{si}}{w(1 + \tau_{si}^l)}$$

Industry s total labor demand is then given by

$$\begin{aligned}L_s &= \sum_{i=1}^{M_s} l_{si} = (1 - \alpha_s) \left(\frac{\nu - 1}{\nu} \right) \sum_{i=1}^{M_s} \frac{p_{si}y_{si}}{w(1 + \tau_{si}^l)} \\ &= (1 - \alpha_s) \left(\frac{\nu - 1}{\nu} \right) \sum_{i=1}^{M_s} \frac{1}{w(1 + \tau_{si}^l)} \frac{p_{si}y_{si}}{P_s Y_s} P_s Y_s \\ &= (1 - \alpha_s) \left(\frac{\nu - 1}{\nu} \right) \frac{P_s Y_s}{\overline{MRPL}_s}\end{aligned}$$

Doing the same for capital, we can rewrite the industry average marginal revenue products as:

$$\begin{aligned}\overline{MRPK}_s &= \alpha_s \left(\frac{\nu - 1}{\nu} \right) \frac{P_s Y_s}{K_s} \\ \overline{MRPL}_s &= (1 - \alpha_s) \left(\frac{\nu - 1}{\nu} \right) \frac{P_s Y_s}{L_s}\end{aligned}$$

Rewriting Y_s as a function of industry level capital and labor $Y_s = TFP_s K_s^{\alpha_s} L_s^{1-\alpha_s}$,

$$\begin{aligned}
TFP_s &= \frac{P_s Y_s}{K_s^{\alpha_s} L_s^{1-\alpha_s}} \frac{1}{P_s} \\
&= \left(\frac{P_s Y_s}{K_s} \right)^{\alpha_s} \left(\frac{P_s Y_s}{L_s} \right)^{1-\alpha_s} \\
&= \left(\frac{\nu}{\nu-1} \right) \left(\frac{\overline{MRPK}_s}{\alpha_s} \right)^{\alpha_s} \left(\frac{\overline{MRPL}_s}{1-\alpha_s} \right)^{1-\alpha_s} \\
&= \overline{TFPR}_s \frac{1}{P_s}
\end{aligned}$$

To measure this variable we will follow [Hsieh and Klenow \[2009\]](#): the elasticity of substitution used is $\nu = 3$, and to measure the wedges they use output distortions and capital relative to labor distortions, a setup which has a one to one mapping to this, but allows a more straightforward interpretation to infer them from the data. The optimization problem is:

$$\pi_{si} = \max_{p_{si}, k_{si}, l_{si}} (1 - \tau_{si}^{*y}) p_{si} (y_{si}) y_{si} - w l_{si} - (1 + \tau_{si}^{*k}) (r + \delta) k_{si}$$

Distortions are measured as:

$$1 + \tau_{si}^{*k} = \frac{\alpha_s}{1 - \alpha_s} \frac{w l_{si}}{(r + \delta) k_{si}}$$

$$1 - \tau_{si}^{*y} = \frac{\nu}{\nu - 1} \frac{w l_{si}}{(1 - \alpha_s) p_{si} y_{si}}$$

We infer large capital distortions if the firm has a ratio of labor to capital that is too high, compared to what you would expect given the industry's labor shares. The model infers high output distortions if the share of labor is low when compared from what one would expect given the industry elasticity of output with respect to labor. To remain as close as possible to [Hsieh and Klenow \[2009\]](#), we trim by 1% the tails of the $TFPR$ and $TFPQ = z$ deviations from industry means measures and the capital and output distortions and then recompute the industry average $TFPR$.

The implied $MRPL$ and $MRPK$ with these wedges are

$$\begin{aligned}
MRPL_{si}^* &\equiv (1 - \alpha_s) \left(\frac{\nu - 1}{\nu} \right) \frac{p_{si} y_{si}}{l_{si}} = \frac{w}{1 - \tau_{si}^{*y}} \\
MRPK_{si}^* &\equiv \alpha_s \left(\frac{\nu - 1}{\nu} \right) \frac{p_{si} y_{si}}{k_{si}} = \frac{1 + \tau_{si}^{*k}}{1 - \tau_{si}^{*y}} (r + \delta)
\end{aligned}$$

These wedges can be made to coincide with the definitions of $MRPL$ and $MRPK$ in the main text (1)-(2) with the redefinitions:

$$1 + \tau_{si}^l = \frac{1}{1 - \tau_{si}^{*y}} \quad 1 + \tau_{si}^k = \frac{1 + \tau_{si}^{*k}}{1 - \tau_{si}^{*y}}$$

G Aggregate TFP Derivation

In order to compare aggregate TFP for different model calibrations, we need to do an analogue of the [Hsieh and Klenow \[2009\]](#) derivation of what is the efficient level of TFP for the case of no labor. We can express aggregate TFP in terms of aggregate $\overline{TFPR} \equiv PY/K$

$$TFP = \frac{Y}{K} = \frac{\overline{TFPR}}{P}$$

P being the usual price index that results from monopolistic competition within the model, and defining $s \in S = \mathcal{K} \times \mathcal{B} \times \mathcal{Z}$

$$P = \left(\sum_{i=a,b,c} \int_S p(y(s,i))^{1-\nu} \mu^i(s) \right)^{\frac{1}{1-\nu}}$$

Using the definition of firm $TFPR \equiv pz$, rewrite this same price index as

$$P = \left(\sum_{i=a,b,c} \int_S \left[\frac{TFPR(s,i)}{z} \right]^{1-\nu} \mu^i(s) \right)^{\frac{1}{1-\nu}}$$

Thus, we can rewrite aggregate TFP as,

$$TFP = \frac{\overline{TFPR}}{P} = \left[\sum_{i=a,b,c} \int_S \left(z \frac{\overline{TFPR}}{TFPR(s,i)} \right)^{\nu-1} \mu^i(s) \right]^{\frac{1}{\nu-1}}.$$

Consumer optimization tells us that firms should equalize $TFPR$, we know that the efficient level of TFP is given when all firms are able to set $TFPR = \overline{TFPR}$, so

$$TFP^e = \left[\sum_{i=a,b,c} \int_S z^{\nu-1} \mu^i(s) \right]^{\frac{1}{\nu-1}}.$$

And define the ratio of actual to efficient TFP as

$$\frac{TFP}{TFP^e} = \left[\sum_{i=a,b,c} \int_S \left(\frac{z}{TFP^e} \frac{\overline{TFPR}}{TFPR(s,i)} \right)^{\nu-1} \mu^i(s) \right]^{\frac{1}{\nu-1}}.$$

Thus, for each parametrization we can compute how close this stationary distribution is close to the efficient one using this formula, and the definitions of aggregate output Y and capital K of equations (17)-(18).